Effect of Quadrupole and Octopole Vibrations of $^{206}$Pb on the $1j_{15/2}$ Neutron Particle State of $^{207}$Pb

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The quadrupole as well as the octopole vibrational states of $^{206}$Pb are coupled to the $2g_{9/2}$, $1f_{11/2}$ and $1j_{15/2}$ neutron states to explain the fragmentation of $1j_{15/2}$ state in $^{207}$Pb as observed in the $^{206}$Pb($d, p$) reaction. The structures of weak fragmented $1j_{15/2}$ states are deduced through the particle-core coupling scheme.

1. Introduction

In order to explain the splitting of the four $1j_{15/2}$ neutron particle states in $^{209}$Pb detected through high resolution $^{208}$Pb($d, p$) reaction [1], we have observed [2] that the effect of core vibration of $^{208}$Pb on neutron particle states ($N=126-184$ and $>184$ shell) is primarily responsible for the weak fragmented $1j_{15/2}$ states of $^{209}$Pb. This core polarisation effect places the zero-order shell model energy of $1j_{15/2}$ at 2.00 MeV with respect to the $2g_{9/2}$ ground state of $^{209}$Pb. It will therefore be interesting to search for the similar $1j_{15/2}$ neutron states of $^{207}$Pb through core-particle coupling scheme.

The neutron states up to 5.692 MeV excitation energy in $^{207}$Pb have been detected in the reaction $^{208}$Pb($d, p$) [3]. From the reaction spectrum, it is observed that some of the two hole one particle states are fragmented. Between the $2g_{9/2}$ state (2.728 MeV) and the $3d_{5/2}$ state (4.389 MeV), only 4.115 MeV state has been identified to be a fragment of $1j_{15/2}$ state with a spectroscopic factor of 0.56 which accounts for only 50% of the shell model strength of this state. We have made an attempt to locate all the fragmented $1j_{15/2}$ neutron particle states in $^{207}$Pb from a core-particle model calculation.

The isobaric analogue resonances studied with the ($p, p'$) reaction [4] have suggested the possible existence of core-particle coupled states in $^{207}$Pb. Also it has been noticed that there exists a correlation between the strength of isobaric analogue resonances corresponding to particle coupled core excited states and the possibility for exciting the core states themselves by direct proton inelastic scattering experiment. The analysis of the resonance structure shows that a model based on a neutron coupled to the $2^+_1$ phonon vibrational state of $^{206}$Pb can successfully account for the gross resonance structure of the ($p, p'$) reaction on $^{208}$Pb. Side by side high resolution ($d, p$) reaction studies [5, 6] also indicate the possible existence of such simple core-particle states above the 2.73 MeV region. The gross structure of the isobaric analogue resonance for the $^{206}$Pb($p, p'$) reaction leading to the $2^+_1$ state of $^{205}$Pb has been found to be altered if the two photon coupling scheme [7] is incorporated into the core-particle model calculation. This definitely accounts for additional fragmentation of the $d_{3/2}$ and $d_{5/2}$ states of $^{207}$Pb which cannot be explained by coupling a particle to the $2^+_1$ state alone. Based on this applicability of the core-particle model to explain the fragmentation of highly excited positive parity particle states in $^{207}$Pb, it will be interesting to investigate a model based on the coupling of the neutron states to the quadrupole and consistently phonon states of $^{206}$Pb. Both the $2^+_1$ and $3^+_1$ states of $^{206}$Pb possess good degree of collectivities as the $B(E2; 2^+_1\rightarrow0^+)$ and $B(E3; 3^+_1\rightarrow0^+)$ values are 6.2 W.U. [8] and 14.2 W.U. [9] respectively.

2. The Core-Particle Model

The Hamiltonian of the system can be taken from Bohr and Mottelson [10],
\[ H = H_{\text{vib}} + H_P + H_{\text{int}}, \]  

where \( H_{\text{vib}} \) gives the vibrational energy of the \(^{206}\text{Pb} \) core, \( H_P \) is the Hamiltonian for the single neutron (\( N = 126-184 \) shell) in the average shell model potential and \( H_{\text{int}} \) represents the core-particle interaction

\[ H_{\text{int}} = - \sum_{\lambda=2}^{3} X_\lambda \hbar \omega_\lambda \sum_{\mu=-\lambda}^{\lambda} [b_{\mu} + (-1)^{\mu} b_{\mu}^*] Y_{\mu \lambda} (\theta, \Phi), \]

where \( X_\lambda \) is the strength of the coupling, \( \hbar \omega_\lambda \) is the energy of the phonon for the \( \lambda \)-mode vibration of the collective core and \( b_{\mu} \) and \( b_{\mu}^* \) represent the usual annihilation and creation operators for the phonons, respectively.

In order to evaluate the eigenvalues for the Hamiltonian (1), we take the wave function for a \( J \) spin state as

\[ \psi_J = \sum_{R_1, R_2} \langle (N_2 R_2, N_3 R_3) R ; (n l_{3/2}) \rangle_{J M}^J, \]

where \( j \) is the angular momentum of the particle, \( R_2 \) and \( R_3 \) are the phonon angular momenta corresponding to the number of quadrupole phonons \( N_2 \) and octupole phonon numbers \( N_3 \), \( R \) is the coupled phonon angular momentum. For our calculation \( N_2 = 1, 2 \) and \( N_3 = 1 \) respectively. In Eq. (3), the summation over \( j \) includes \( 2g_{9/2}, 1i_{11/2} \) and \( 1j_{15/2} \) neutron states which satisfy the relation \( J = R + j \). The matrix elements of \( H_{\text{int}} \) in this representation are given by

\[ \langle N_2 R_2, N_3 R_3 ; j | H_{\text{int}} | N_2 R_2', N_3 R_3' ; j' \rangle = \sum_{k} X_\lambda \hbar \omega_\lambda \sum_{\mu=-\lambda}^{\lambda} \left[ b_{\mu} + (-1)^{\mu} b_{\mu}^* \right] Y_{\mu \lambda} (\theta, \Phi), \]

where the symbols used have their usual meaning. The subscripts \( \eta \) in \( N_\eta \) and \( R_\eta \) assume the value 2 when \( \lambda = 3 \) and vice-versa. The diagonal matrix elements of \( H \) (1) are the sum of the particle energy \( \varepsilon_j \) and the energy of the core \( E_c \). \( E_c \) is given by

\[ E_c = N_2 \hbar \omega_2 + N_3 \hbar \omega_3 \]

3. Results

The \( 2g_{9/2}, 1i_{11/2} \) and \( 1j_{15/2} \) states of \(^{207}\text{Pb} \) have been coupled with the quadrupole one and two and the octupole one phonon states of \(^{206}\text{Pb} \). Using (1), (3), (4) and (5) the matrix of \( H \) for \( 15/2^- \) has been set up. The single particle energies for the \( 2g_{9/2} \) and \( 1i_{11/2} \) states have been taken from shell model level positions with respect to the \( 3p_{1/2} \) state of \(^{207}\text{Pb} \) [3]. The energies of \( 2_1^- \) and \( 3_1^- \) states have been taken 0.803 MeV and 2.615 MeV respectively [8]. The energies of \( J = 15/2^- \) states have been extracted by diagonalization. Only the parameters \( \varepsilon_{ij_{15/2}}, X_2 \) and \( X_3 \) have been optimized to locate the primary \( 15/2^- \) state of appreciable spectroscopic factor. The results of the present calculation have been listed in Table 1. Apart from fitting the main \( j_{15/2} \) state at 4.043 MeV, we are able to locate two strong (4.941 MeV and 5.767 MeV) and two weak (6.172 MeV and 6.675 MeV) \( j_{15/2} \) states of \(^{207}\text{Pb} \). The configurations for the 4.941 MeV and 5.767 MeV states come primarily through \( 2g_{9/2}, j_{15/2} \) and \( 3_1^-, g_{9/2} \) collective states whereas the 6.172 MeV and 6.675 MeV states arise from \( 3_1^-, i_{11/2} \) and \( 4_2^+, j_{15/2} \) collective configurations. It is possible that a few of the unassigned states above 4 MeV excitation in the experimental data [3] may belong to these predicted fragments of the \( 15/2^- \) state.

4. Conclusion

In view of the calculated results on \( j_{15/2} \) particle state of \(^{207}\text{Pb} \), the following points emerge.