A Parametrized S-Matrix Model of Deep-Inelastic Scattering

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Deep-inelastic collisions are studied within a phenomenological approach based on a parametrized S matrix. The importance of a correct treatment of the Poisson sum is emphasized and the possibility of non-zero angular-momentum transfer is included. Statistical fluctuations are also considered. It is shown that the spin polarization is extremely sensitive to the treatment of the Poisson sum and that the spin alignment is independent of the scattering angle in the absence of statistical fluctuations.

1. Introduction

Deep-inelastic collisions (DIC) between heavy ions are rather complex and, even though some microscopic theories have been developed (see, for example, Ref. 1), there is still the need for a phenomenological 'bridge' between data and basic theory. Such a bridge is available in the approach based on a parametrized S matrix [2–4, 17].

The appeal of this approach is that it is simple and readily usable, since analytic expressions can be derived for observable quantities such as the double-differential cross section [2, 17] and spin polarization [3]. It can also account for some structure observed in the energy spectra of products resulting from DIC induced by identical heavy ions [4].

This model is, however, incomplete in at least two respects. Firstly, a one-term approximation is made to the well-known Poisson sum (see, for example, Ref. 5). While this is an excellent approximation in the absence of orbiting, we will show in this paper that for high energy losses the effect of other terms is not always negligible, due to the presence of dispersion [2, 6]. The possible relevance to DIC of the full Poisson sum, i.e. orbiting terms, is not yet settled and has been discussed by Dietrich et al. [18]. They conclude that for partially-damped events the cross section is insensitive to the orbiting terms, but that in the strongly-relaxed limit the full Poisson sum leads, as expected, to a simple 1/sin θ dependence of the cross section.

Secondly, no spin transfer is explicitly considered in Hartmann's model, which means that only an estimate can be given of the spin polarization [3, 6], and that the spin alignment cannot be treated at all. Although some authors [18] have not neglected the intrinsic angular momenta, and hence the possibility of spin transfer, neither the spin polarization nor the spin alignment has been calculated in this approach. The model to be presented here will be used to calculate both of these quantities.

A crucial feature of Hartmann's model is the inclusion of quantal, dynamic [17] and statistical fluctuations [2], which enables the observed decrease [7] in the magnitude of the spin polarization at high energy losses to be qualitatively described [3]. These dispersive effects, the causes of which have been described elsewhere [2, 6], have also been studied with the time-dependent Hartree-Fock method (TDHF) [8]. Although standard TDHF cannot account for fluctuations because of its mean-field nature, the inclusion of a distribution of initial conditions makes this possible, since there is then a distribution of final states too [8].

A similar idea to this has been used by Lee et al. [6] in a phenomenological study of DIC. They showed that the treatment of statistical fluctuations can be reduced to a consideration of many classical trajec-
ories, each of which leads to the same scattering angle. These different contributions are weighted, according to a Lorentzian distribution, relative to a 'mean' trajectory, and the form of this distribution function is justified [6]. In practice the inclusion of statistical fluctuations with this method is easier than with that given elsewhere [2]. However, dynamical fluctuations [17] were neglected in [6], which is a poor approximation for the highest energy losses and for heavy systems. This approach explicitly includes spin transfer, enabling a good description to be given of the spin polarization in DIC between 'light' heavy ions [6].

In the present paper we modify the model of Hartmann [2] to include all necessary Poisson terms and also spin effects. Statistical fluctuations are treated using the method of Lee et al. [6], and, for the first time, the spin alignment is treated in such a phenomenological approach. In Sect. 2 we describe the model, which is applied to the DIC $^{40}$Ar+$^{232}$Th at $E_{lab}=388$ MeV in Sect. 3. We end with our conclusions.

2. The Model

We present here a summary of our model: further details can be found elsewhere [2, 6].

We assume that direct-reaction theory is applicable (since nuclear identities are approximately maintained in DIC) and parametrize the transition form factor as [6, 19]

$$I_{l' l} = f(l' - L) g(l - l')$$

(1)

where $f$ and $g$ are the 'diffractive' and 'matching' terms, respectively. The form of (1) has been justified elsewhere [19]. By using a quantization axis perpendicular to the scattering plane, the scattering amplitude for a transition from a spin-zero state to one with spin $(I, M)$ can be written [6]

$$\beta_{IM}(\theta) \propto d_{0M}^{-1/2} \{g(M) F^+(\theta) + g(-M) F^-(\theta)\}$$

(2)

With a Gaussian parametrization of $f(l' - L)$ [2], $F^\pm(\theta)$ are given by

$$F^\pm(\theta) \approx \frac{\alpha(Q)}{\sqrt{2\pi} \chi} \sum_{n=-\infty}^{\infty} \exp\left[2\pi i n E L \right]$$

$$\times \exp\left[-\beta(Q)(\Theta \pm \theta + 2\pi n)^2\right]$$

(3)

(apart from an uninteresting phase), where

$$\alpha(Q) \equiv 2^{1/4} (L A / \xi)^{1/2}$$

(4a)

$$\beta(Q) \equiv [2 + i A^2 \Theta'] / [4 \xi^2]$$

(4b)

In these equations the initial (final) angular momentum is $l'(l)$, and $L$ is the grazing angular momentum in the final channel. The width of the distribution $f(l' - L)$ is $A$, and the $Q$ dependence of the quantities above comes from the relationship between $L$ and energy loss.

In obtaining (2) and (3) the usual approximations for heavy-ion scattering have been made [2, 5, 6], except that we have kept all the Poisson terms in (3). If spin effects are neglected ($I=M=0$) and only the $n=0$ Poisson term retained our expressions reduce to those of Hartmann [2]. It is easy to see that at high energy losses the $n \neq 0$ terms become important, since then $\xi$ becomes large [2] and $\beta(Q)$ correspondingly small. Consequently even non-zero values of $n$ can give a contribution that is not negligible.

The amplitude of (2), which we may term 'diffractive' [6], is that for a transition to a specific final state, but in DIC the high level density prevents one such state being resolved experimentally. The effects of energy averaging (i.e. of statistical fluctuations) must therefore be included. A particularly simple treatment of these fluctuations has been given, with theoretical justification, elsewhere [6]. The average ('coarse-grained') cross section may be written in this formalism as

$$\langle \sigma_{Q} \rangle \propto \int dQ' H(Q' - Q) A(Q')$$

$$\cdot [\|F^+(Q', Q')\|^2 + |F^-(Q', Q')|^2],$$

(5)

where

$$H(x) = \frac{\Gamma/2\pi}{x^2 + \frac{1}{4}\Gamma^2}$$

(6)

and

$$A(Q) \equiv \sum_{IM} |d_{0M}^{-1/2} g(M)|^2 G(I).$$

(7)

Here $G(I)$ is the level density of states of spin $I$ and $\langle \ldots \rangle$ in (5) (and throughout this paper) indicates that the quantity is evaluated with the inclusion of statistical fluctuations. In (6) $\Gamma$ is the averaging interval (see below), and we note that in writing (5) we have neglected any interference between $F^+$ and $F^-$. This is justified for high level densities.

The simplicity of this approach to statistical fluctuations [6] is that only one extra quantity, $\Gamma$, is