Mode Conversion in Optical Second Harmonic Generation: Theory and Experiment

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Kingston and McWhorter's theory of mode conversion in second harmonic generation with a thin crystal [1] has been extended to include all positions of the crystal relative to the focus of the fundamental beam. Phase differences between the harmonic modes are predicted which depend upon the crystal position. Experiments to test these predictions have been performed with a 1.15 μm helium-neon laser and non-critical phase-matching in lithium niobate. Results are presented for a number of fundamental modes.

1. Introduction

The problem of transverse mode conversion in optical second harmonic generation is one which has had very little experimental study. Theoretical calculations are available in the form of Kingston and McWhorter's paper [1], which is only valid for thin crystals, and Asby's [2-4] complete theory for crystals without double refraction. The only relevant experimental results published are the early ones of Ashkin et al [5] on the relative conversion efficiencies for several fundamental transverse modes and a mention in a paper from our laboratory [6] of a particular example of harmonic generation from a TEM_{10} mode. In this paper we present new experimental results concerning the form of the harmonic beam for a number of fundamental transverse modes and relate the results to an extension of Kingston and McWhorter's theory.

The original analysis of Kingston and McWhorter [1] was restricted to thin crystals placed in the focal plane of the fundamental beam. The first part of this paper extends their approach to include movement of the thin crystal away from the focus. This has the effect of changing the phase differences between the second harmonic modes and thereby giving a distance-dependent harmonic field pattern. The second part of the paper covers the results of an experiment using a crystal of lithium niobate which confirms the theoretical predictions. The absence of double-refraction in lithium niobate when phase-matching normal to the optic axis (non-critical matching) makes its use in this experiment a particular advantage since the spatial distribution of the harmonic field retains a form similar to the harmonic polarisation in the crystal.

2. Theory

It is well known that spherical mirror lasers oscillate on one or more stable TEM modes. Complete expressions for the fields of these modes were first derived by Boyd and Gordon [7] for the particular case of a confocal resonator. Their expressions can also be used for the refocused beam from a laser through the definition of an equivalent confocal parameter b.

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Figure 1 Beam geometry for harmonic generation with thin crystal ($l << b$).

(In what follows we use $z_0 = b/2$, a choice of no significance other than that it makes the field expressions a little simpler.) A dependent parameter is $W_0 (= \sqrt{\beta/k})$ which determines the transverse extent of the beam at the focus. In the following analysis the fundamental modes will be labelled $\text{TEM}_{mn}$ and the generated harmonic modes $\text{TEM}_{2s2s}$. As shown by Kingston and McWhorter [1] and Asby [2], $r$ can take up the integral values 0 to $m$ and $s$ values 0 to $n$. The harmonic modes are thus always of even order. The geometry assumed for the calculation is indicated in fig. 1.

We start then with the field of the fundamental written in terms of the functions introduced by Boyd and Gordon:

$$E_1 = \frac{E_{10} N_{m,n}}{[1 + (z/z_0)^2]^{1/2}} \cdot H_m \left[ \frac{2}{W_{10}^2 (1 + (z/z_0)^2)} \right]^{1/2} \cdot H_n \left[ \frac{2}{W_{10}^2 (1 + (z/z_0)^2)} \right]^{1/2} \cdot \exp \left[ \frac{(x^2 + y^2)}{W_{10}^2 (1 + (z/z_0)^2)} \right] \cdot \exp i \left[ \omega t - k_1 z - k_{1z} \left( \frac{x^2 + y^2}{2z_0^2 (1 + (z/z_0)^2)} \right) - (m + n + 1) \tan^{-1}(z/z_0) \right].$$

(1)

where $H_m$ and $H_n$ are Hermite polynomials of orders $m$ and $n$ respectively and $k_1$ is the propagation constant at the fundamental frequency. The above expression differs from equation 20 of Boyd and Gordon's paper [7] in that the phase reference has been chosen as the $z = 0$ plane, and the normalisation constant $N_{m,n}$ is such that the total power flow in a $\text{TEM}_{mn}$ mode is equal to 1. $N_{m,n}$ is given by:

$$N_{m,n} = \frac{2}{E_0 W_0} \left[ \pi \sqrt{\frac{\epsilon}{\mu_0}} \cdot 2^m \cdot 2^n \cdot m! n! \right]^{-1/2}.$$  

(2)

(A full derivation of the normalisation constant is given in the appendix.)

Following Kingston and McWhorter, the field changes that result from the parametric coupling between three modes (labelled $a$, $b$ and $c$) are described in terms of small perturbations of the mode amplitude factors $u_0(z)$ etc., where

$$E_a \text{ etc.} = \text{Re}[u_a(z)E_{a0}(x, y, z) \exp i (\omega_a t - k_a z (1 + \beta))];$$  

(3)

$E_{a0}(x, y, z)$ is that part of the mode field containing terms slowly varying with $z$, and $\beta = (x^2 + y^2)/(2z_0^2 (1 + (z/z_0)^2))$. 

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