GUIDES WITH FLAT SPRINGS FOR FORWARD DISPLACEMENTS

Ya. M. Tseitlin

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The quality of guides for the movement of measuring surfaces determines to a considerable extent the metrological indexes and the reliability of operation of precision instruments and automatic testers. These devices are provided on a wide scale with flat spring guides (spring suspensions), which are free from play and external friction, characteristic of sliding and rolling guides. Spring guides are highly stable, reliable, have a long life, simple design and are easy to manufacture. However, it is wrong to ignore certain spring-guide characteristics, since failure to account for them leads to incorrect designs resulting in overloading and premature failures.

Below we deal only with widely-known spring guides of the parallelogram type for forward displacements. The designer of such guides is faced either with the problem of determining the deviation of a carriage suspended on flat springs from a given forward movement, or the reverse problem of determining the basic parameters of the suspension for a given tolerated deviation from the forward movement. The above deviation can be determined quantitatively by the slope angle of the carriage in the plane and within the range of its movement to the position it occupied originally (see θ in Fig. 2). It is often also necessary to determine the value of the transverse ("stray") displacements λ of the carriage for a given longitudinal displacement f, the maximum stress σ_{max} in the flat springs, the required value of the propelling force H (or its equivalent measuring effort) etc.

The evaluation of θ and λ can be considered as accuracy computations. The behavior of a spring parallelogram should be examined under actual load conditions by taking into consideration all the main external forces. Insufficient attention has been paid to this problem in our technical publications. We only know of works [1, 2] in which, however, there are no accuracy computations for spring guides.

Considerably greater attention is paid to spring parallelograms in foreign technical literature [3-9]. There is also a good bibliography of works published on this subject up to 1954 [10].

Accuracy computations are provided in the works of Plainevaux [3-7], but his design formulas do not take into account the effect of friction on the measuring surface of the moving carriage and the variation in the spatial position of the suspension on the accuracy of the guiding device. These problems are of immediate interest for a large number of lay-on gear gauges in the course of whose operation and setting changes may occur in the direction of the friction forces on the measuring surfaces and in the spatial position of the instrument. This in turn can lead to supplementary errors and raise the instability of measurement.

In order to facilitate further explanations it is necessary to refer to the basic relationships obtained by Plainevaux for approximate solutions with the assumption of small deformations in the flat springs. A precise solution for any deformation is obtained in the form of a system of equations containing elliptical integrals.

On the basis of the well-known differential equation of a bent (originally flat) elastic rectangular strip (Fig. 1)

\[ EJ \frac{d^4X}{dZ^4} + PX = M_0 - FZ. \]  

(1)

Plainevaux obtained the following approximate design relationships for determining the bending moment \( M_A \) at the moving end of the spring, the transverse component \( F \) of the external bending force \( Q \),
the bending moment \( M_0 \) at the fixing end, the lowering \( \lambda \) of the spring's moving end in terms of its displacement \( f \), the turning angle \( \Theta \) of the spring with respect to its initial direction in its neutral position, and the longitudinal component \( P \) of the external force \( Q \)

\[
\frac{M_0}{EJ} = 4 \tan \Theta \left[ 1 + \frac{1}{30} \frac{PL^2}{EJ} \right] - \frac{6}{L} \left[ 1 + \frac{1}{60} \frac{PL^2}{EJ} \right],
\]

\[
\frac{P}{EJ} = -6 \tan \Theta \left[ 1 + \frac{1}{60} \frac{PL^2}{EJ} \right] + \frac{12}{L} \left[ 1 + \frac{1}{10} \frac{PL^2}{EJ} \right],
\]

\[
\frac{M_a}{EJ} = -2 \tan \Theta \left[ 1 + \frac{1}{60} \frac{PL^2}{EJ} \right] + \frac{6}{L} \left[ 1 + \frac{1}{10} \frac{PL^2}{EJ} \right],
\]

\[
\frac{\lambda}{L} = \frac{3}{5} \left( \frac{f}{L} \right)^2 \cdot \left[ 1 + \frac{1}{420} \frac{PL^2}{EJ} \right] - \frac{1}{10} \frac{f}{L} \tan \Theta \left[ 1 + \frac{1}{70} \frac{PL^2}{EJ} \right].
\]

In (1) and (2) \( L \) is the length of the flat spring in the neutral position; \( E \) is Young's modulus, \( J \) is the moment of inertia of the spring's cross section with respect to its neutral axis.

The top signs should be used for a compressive, and the bottom signs for a tensile longitudinal force.

Relationships (2) only hold for the condition that

\[
\frac{PL^2}{EJ} \ll 1.
\]

In the majority of cases when spring suspensions are used in instruments and automatic machines this condition (a small longitudinal force) is fulfilled, since the weight of the moving carriage is normally small. Moreover, in deriving (2) the terms with \((\tan \Theta)^2\) have been neglected in view of the small slanting angle of the carriage and, hence, a small value of \( \tan \Theta \) as compared with unity.

Computations (2) provide very satisfactory results for the relative deformations \( f/L \) in the range of \( f/L \leq 0.2 \) (the error does not exceed 5-10%).

Let us now compute the spring parallelogram.

Figure 2 shows a parallelogram with an arbitrary angle between base 1 and the horizontal direction (angle \( \psi \) can assume any value). The position of the known external forces \( V, N \) and \( G \) (the weight of the carriage) which act on carriage 2 has been taken as approximately the same as in an existing instrument. Friction force \( P_f \) is directed along the measuring jaw.

It is required to determine measuring pressure \( H \) and the slope angle \( \Theta \) of the carriage within the range of its travel.

We derive a system of equations for a balanced position of carriage 2

\[
P + P' = G \cdot \cos \psi \pm P_{r} \cdot \cos \Theta,
\]

\[
\pm P_f \cdot \sin \Theta \mp V \cos \Theta + P + P' + N + G \sin \psi = H,
\]

\[
H \cdot a + M_A + M_A' + G \cdot \cos \psi (v \cos \Theta + h \sin \Theta) +
\]

\[
N \cdot m' \cdot \cos \Theta + P' \cdot S \cdot \cos \Theta = V \cdot e - G \sin \psi (h \cos \Theta - v \sin \Theta) -
\]

\[
- P' \cdot S \cdot \cos \Theta \pm P \cdot r = 0,
\]

where \( P, P', F \) and \( F' \) are the longitudinal and transverse components of the reaction of springs 3 and 4; \( v \) and \( h \) are the coordinates of the carriage's center of gravity.

By neglecting the terms of the fourth and higher orders of magnitude with respect to \( f/L \), we obtain from (2) and (3) after certain transformations the following equations:

\[
H = V + N + P_f (\pm \sin \Theta) + 24 \frac{f}{L} \left[ \frac{EJ}{L^5} \mp \frac{1}{20} (G \cos \psi + P_{f}) \right] - 12 \tan \Theta \left[ \frac{EJ}{L^5} \mp \frac{1}{120} (G \cos \psi + P_{f}) \right] \pm G \sin \psi;
\]

(4)