1. Introduction

The concept of a relation (or set of ordered pairs) is one of the unifying ideas in mathematics and should therefore be utilized in its teaching. We will, in this article, discuss some examples of how relations can be used in the teaching of probability theory. More specifically, we will discuss how the very important concept of independence of events can be taught in the language of relations. We will also discuss the treatment of such experiments where the set of outcomes is a relation. We have used these ideas in an elementary textbook (grades 10–12) in probability theory within the Elements of Mathematics Program of the Comprehensive School Mathematics Program (CSMP), see [1], [4] and [5]. Preceding this textbook are two books giving a detailed treatment of relations and functions, [2] and [3].

2. Basic Concepts of Probability Theory

In this section we give a short review of some fundamental concepts in probability theory without going into a discussion of the practical background which has motivated the definition of these concepts. The basic building block in this theory is the concept of a probability space \((\Omega, \mathcal{A}, P)\). Here \(\Omega\) is a set, the set of outcomes or the outcome set, \(\mathcal{A}\) is the set of events, all of which are subsets of \(\Omega\), and \(P\) is a probability measure. We restrict our discussion to the case when \(\Omega\) is a finite set. In this case one can take \(\mathcal{A}\) to be the power set \(\mathcal{P}(\Omega)\), the set of all subsets of \(\Omega\). One can then delete the second component in \((\Omega, \mathcal{A}, P)\). In passing, we remark that one can more generally start with axioms for set \(\mathcal{A}\) of events and in doing this from the very beginning use the concept of an implication relation. For details of such a treatment, see Hennequin [6].

Thus one can start with the concept of a finite probability space \((\Omega, P)\). Here the outcome set \(\Omega\) is a finite set and the probability measure \(P\) is a real-valued function with domain \(\mathcal{P}(\Omega)\) such that

\[
\begin{align*}
(1) & \quad 0 \leq P(A) \leq 1 \quad \text{for each } A \in \mathcal{P}(\Omega) \\
(2) & \quad P(\Omega) = 1 \\
(3) & \quad A \cap B = \emptyset \Rightarrow P(A \cup B) = P(A) + P(B) \quad \text{for all } A, B \in \mathcal{P}(\Omega).
\end{align*}
\]
In connection with finite probability spaces, the concept of a point probability function (ppf) is quite useful. If \((\Omega, P)\) is an fps its ppf is defined as the real-valued function \(p\) with domain \(\Omega\) such that

\[
p(w) = P(\{w\}) \quad \text{for each } w \in \Omega.
\]

It can be easily shown that for this function \(p\) the following holds:

\[
p(w) \geq 0 \quad \text{for each } w \in \Omega \quad \text{and} \quad \sum_{w \in \Omega} p(w) = 1.
\]

Furthermore, one can prove that if \(\Omega\) is a finite set and \(p\) a function such that

\[
\begin{align*}
(4) \quad & p: \Omega \rightarrow R \\
(5) \quad & \sum_{w \in \Omega} p(w) = 1,
\end{align*}
\]

then there exists one and only one fps \((\Omega, P)\) such that \(p\) is the ppf of \((\Omega, P)\). In this case the probability measure is determined by

\[
(6) \quad P(A) = \sum_{w \in A} p(w) \quad \text{for each } A \in \mathcal{P}(\Omega).
\]

This implies that an fps \((\Omega, P)\) is uniquely determined by giving a finite set \(\Omega\) and a function \(p\) with properties (4) and (5). Figure 1 gives two examples of fps’s determined in this way.

![Fig. 1.](image)

3. The binary independence relation

We start with a practical situation. A bag contains green and red cubes and balls. The number of the four different kinds of objects (green cubes, red cubes, green balls, red balls) are given in the following table.