Lasing without inversion

II. Raman process created atomic coherence

S.Y. Zhu 1, M.O. Scully 2, H. Fearn 1, and L.M. Narducci*

1 Center for Advanced Studies and Department of Physics and Astronomy, University of New Mexico, Albuquerque, NM 87131, USA
2 Max-Planck Institut für Quantenoptik, W-8046 Garching, Federal Republic of Germany

Received 1 July 1991

Abstract. We develop a nonlinear theory of lasing without inversion in a four-level atomic system. The appearance of lasing without inversion is the result of atomic coherence induced between two lower levels by a Raman process. We consider both nondegenerate and degenerate configurations and obtain expressions for the linear gain and the laser output intensity. We identify the conditions for ideal lasing without inversion.

PACS: 42.50.Md; 42.55. - f; 32.80. - t

I. Introduction

In a previous paper [1] we have studied the process of lasing without inversion in open and closed systems, for both nondegenerate and degenerate configurations (open systems are such that the lasing levels can decay to other lower lying states, while in closed systems the decay processes are confined to the active levels of interest; the adjectives nondegenerate and degenerate refer to the amplification of a pair of distinct laser lines or of a single frequency component). The realization of lasing without inversion is the consequence of the appearance of atomic coherence between the two lower levels which reduces the rate of absorption for laser light. In the open system configuration, the atomic coherence can be achieved through a suitable initial preparation before the atoms enter the cavity. In the closed system, we assumed that the atoms developed coherence between the two lower levels in the absence of a laser field. This coherence can be achieved through a Raman process or by the interactions with a microwave field. In this paper we focus on the Raman process-created coherence, and in a subsequent paper we will study the microwave-induced coherence.

It is known that a resonant interaction between a three-level A system and one (or two) strong electromagnetic field(s) (Raman processes) can lead to atomic population trapping in a linear combination of the lower levels [2–6] and consequently leads to the reduction or even exclusion of transitions from the lower levels to a higher lying level. The trapping is due to the electromagnetic field-induced atomic coherence between the two lower states. With zero absorption by the lower levels, the possibility of laser emission from the upper level down to the two lower levels is enhanced. In this paper we show how to use the Raman processes to create the atomic coherence, and to induce lasing without inversion in a four-level closed system. We also establish the necessary conditions for lasing without inversion and the relation between the laser intensities and the intensities of the Raman fields.

In a previous analysis limited to the linear regime, [7] we have shown that lasing without inversion (resulting from complete or partial cancellation of absorption from the lower-levels) is possible also with the help of the A quantum-beat laser scheme, even in the collision-dominated regime. More recent work has shown that a double A-system may also display gain without inversion at optical transitions [8]. The nonlinear analysis of [8] is conceptually identical to the Raman case considered in this work. Here, however, we wish to account for the effects of the off-diagonal atomic density matrix element $\rho_{ad}$ which couples the upper level $|a\rangle$ to the auxiliary Raman level $|d\rangle$. For this reason we do not neglect this contribution to the atomic polarization.

We consider lasing without inversion in the four-level closed systems shown schematically in Fig. 1. The transitions between the upper level $|a\rangle$ and the two lower levels $|c\rangle$ and $|b\rangle$ produce two fields in the nondegenerate case or one field in the degenerate case. The two lower levels are coupled by Raman processes through an auxiliary level $|d\rangle$. The unperturbed Hamiltonian for the atomic system is

$$H_0 = \sum_{j=1}^{N} \sum_{s=a}^{d} \hbar \omega_s |s\rangle \langle s|,$$

(1)
II. The nondegenerate configuration

A. Equations of motion

The interaction Hamiltonian is comprised of two parts,

\[ V = V_1 + V_2 \]

where \( V_1 \) denotes the interaction between the laser fields and the atoms, and \( V_2 \) the interaction between the input Raman fields and the atoms. The two Raman fields couple level \( |d\rangle \) to levels \( |c\rangle \) and \( |b\rangle \), as shown in Fig. 1a and induce a Raman coherence between them. Each Raman field (or laser field) couples with only one atomic transition due to some selection roles. The explicit forms of these interaction energies are given by

\[ V_1 = \sum_{j=1}^{N} \hbar [g_j E_j e^{-i\omega_j t} |a\rangle \langle c| + g_2 E_2 e^{-i\omega_2 t} |a\rangle \langle b|] + H.A., \] (2a)

\[ V_2 = \sum_{j=1}^{N} \hbar [\alpha e^{-i\omega_j t} |d\rangle \langle c| + \beta e^{-i\omega_j t} |d\rangle \langle b|] + H.A., \] (2b)

where \( \alpha \) and \( \beta \) are the Rabi frequencies of the Raman fields whose carrier frequencies are \( \omega_3 \) and \( \omega_4 \), respectively. \( E_1 \) and \( E_2 \) are the lasing field strengths with carrier frequencies \( \omega_1 \) and \( \omega_2 \), respectively, \( g_1 \) and \( g_2 \) are the coupling constants between the laser fields and the corresponding transitions. The two laser fields and the Raman fields, in the ring cavity, propagate in the same directions with wave vectors \( k_1, k_2, k_3 \) and \( k_4 \), respectively. We assume that the phase matching condition is fulfilled \((k_1 - k_2 - k_3 + k_4)L \ll 1\), where \( L \) is the cavity length. The equations of motion for the density matrix elements of the total atomic system are

\[ \dot{\rho}_{dd} = -\frac{1}{T_4} \rho_{dd} + i[a^* e^{i\omega_4 t} \rho_{dc} - \alpha e^{-i\omega_3 t} \rho_{cd} + \beta^* e^{i\omega_4 t} \rho_{db}] - \beta e^{-i\omega_4 t} \rho_{bd}, \] (3a)

\[ \dot{\rho}_{aa} = -\frac{1}{T_4} \rho_{aa} + \lambda (\rho_{bb} + \rho_{cc}) + i[g_1^* E_1 e^{i\omega_4 t} \rho_{ac}] - g_1 E_1 e^{-i\omega_1 t} \rho_{ca} + g_2^* E_2 e^{i\omega_3 t} \rho_{ab} - g_2 E_2 e^{-i\omega_3 t} \rho_{ba}, \] (3b)

\[ \dot{\rho}_{bb} = \frac{1}{2T_1} \rho_{aa} + \frac{1}{2T_1} \rho_{dd} - \lambda \rho_{bb} - \frac{1}{\tau_1} (\rho_{bb} - \rho_{cc}) + i[g_2^* E_2 e^{-i\omega_3 t} \rho_{ba}] - g_2^* E_2 e^{i\omega_3 t} \rho_{ab} + \beta e^{-i\omega_3 t} \rho_{ba} - \beta^* e^{i\omega_3 t} \rho_{db}, \] (3c)

\[ \dot{\rho}_{cc} = \frac{1}{2T_1} \rho_{aa} + \frac{1}{2T_1} \rho_{dd} - \lambda \rho_{cc} - \frac{1}{\tau_1} (\rho_{cc} - \rho_{bb}) + i[g_1^* E_1 e^{-i\omega_1 t} \rho_{ca}] - g_1^* E_1 e^{i\omega_1 t} \rho_{bc} + \alpha e^{-i\omega_3 t} \rho_{cd} - \alpha^* e^{i\omega_3 t} \rho_{dc}, \] (3d)

\[ \dot{\rho}_{ba} = -\frac{1}{T_2} \rho_{ba} + i\omega_{ab} \rho_{ba} - i[g_1^* E_1^* (\rho_{aa} - \rho_{bb}) e^{i\omega_4 t}] - g_1^* E_1^* e^{i\omega_4 t} \rho_{bc} + \beta^* e^{i\omega_3 t} \rho_{da}, \] (3e)

\[ \dot{\rho}_{cb} = -\frac{1}{T_2} \rho_{cb} + i\omega_{bc} \rho_{cb} - i[g_2^* E_2^* (\rho_{aa} - \rho_{bb}) e^{i\omega_4 t}] - g_2^* E_2^* e^{i\omega_3 t} \rho_{dc} + \beta e^{i\omega_3 t} \rho_{da}, \] (3f)

\[ \dot{\rho}_{ad} = -\frac{1}{T_2} \rho_{ad} + i\omega_{ac} \rho_{ad} - i[g_2^* E_2^* (\rho_{aa} - \rho_{bb}) e^{i\omega_3 t}] - \beta^* e^{i\omega_4 t} \rho_{db} + \beta e^{i\omega_4 t} \rho_{da}, \] (3g)

\[ \dot{\rho}_{bd} = \frac{1}{T_2} \rho_{bd} + i\omega_{bd} \rho_{bd} - i[(\rho_{dd} - \rho_{bb}) \beta^* e^{i\omega_3 t}] - \alpha e^{i\omega_4 t} \rho_{bc} + \alpha^* e^{i\omega_4 t} \rho_{db}, \] (3h)

\[ \dot{\rho}_{cb} = -\frac{1}{T_2} \rho_{cb} + i\omega_{bc} \rho_{cb} - i[g_1^* E_1^* (\rho_{aa} - \rho_{bb}) e^{i\omega_4 t}] - g_2^* E_2^* e^{i\omega_3 t} \rho_{bc} - g_2 E_2 e^{-i\omega_3 t} \rho_{cb}, \] (3i)

where we have included incoherent pump contributions from levels \( |a\rangle \) and \( |b\rangle \) to level \( |a\rangle \), with the same rate \( \lambda \), \( T_1 \) (\( T_2 \)) and \( \tau_1 \) are population lifetimes from the upper level \( |a\rangle \) (the auxiliary level \( |d\rangle \)) to the lower levels \( |c\rangle \) and \( |b\rangle \), and between the two lower levels, respectively, and the corresponding polarization lifetimes are \( T_2 \) (\( T_2 \)) and \( \tau_2 \). Here \( \omega_{ab} = = \omega_a - \omega_b \) where \( \omega_a, \omega_b, a, b, c, d \).

We introduce the slowly varying functions of time \( \tilde{\rho}_{ab} \) defined by

\[ \rho_{ca} = \tilde{\rho}_{ca} e^{i\omega_4 t}, \] (4a)

\[ \rho_{ba} = \tilde{\rho}_{ba} e^{i\omega_3 t}, \] (4b)