Unitarity Bounds on Higgs Boson Masses in the Weinberg–Salam Model with Two Higgs Doublets

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Abstract. In the Weinberg–Salam model with two Higgs doublets upper bounds are derived for the Higgs boson masses by applying partial wave unitarity to the tree graphs of Higgs–Higgs scatterings. The upper bounds we find are \( m_{h^+} < 1.7 \text{ TeV}, m_{h^0} < 1.4 \text{ TeV}, m_{h^0} < 1.4 - 2.2 \text{ TeV} \).

1. Introduction

By now the experimental status of the standard Weinberg–Salam model of weak and electromagnetic interactions is excellent, although \( Z, W^\pm \) are not yet discovered, and also the precise nature of the Higgs sector is to be clarified. The experimental fact that \( m_Z \cos^2 \Theta_W/m_W^2 \approx 1 \) suggests the presence of Higgs doublets. The simplest case of one Higgs doublet has been extensively discussed (see e.g. [1–3]). Unfortunately, in this case the Higgs-fermion coupling is suppressed by the factor \( m_f/m_W \) which makes the identification of a single neutral Higgs boson difficult. It was, however, recognized soon [4–7, 3] that including charged Higgs bosons may be advantageous. While more than one Higgs doublet is not excluded experimentally, charged Higgses could be seen more easily. In this case there may also exist somewhat enhanced Higgs-fermion couplings [6]. Indeed, since more than one non-zero vacuum expectation values are present, one can easily arrange that fermions and vector bosons get masses from different scales. If one of the vacuum expectation values, \( v_2 \), giving rise to the fermion mass is smaller than the other, then a Higgs-fermion coupling proportional to \( m_f/v_2 \) is generated which is larger than the one in case of one Higgs boson. However, the ratio of the vacuum expectation values cannot be optional, since the experimental value of the \( K_L - K_S \) mass difference severely restricts the magnitude of charged Higgs effects [8].

In case of many Higgs bosons the price to be paid is the large number of parameters. For two Higgs doublets discussed in this paper we introduce five physical Higgs particles: \( H^\pm, H^0, \phi^0, h^0 \) with unknown masses and coupling constants. As is known, the Weinberg–Linde lower bound of 7.1 GeV [1] is valid for the heaviest Higgs boson. For one neutral Higgs boson an upper bound of 1 TeV exists for the Higgs mass [9, 10].

The task of this paper is to derive upper bounds for the five Higgs masses in the two Higgs doublets. This is carried out in a model general enough, which was discussed from the point of view of enhanced \( f-H \) couplings in [6]. A brief description of the model is provided in Sect. 2.

In Sect. 3 the transitions Higgs + Higgs → Higgs + Higgs are discussed in tree graph approximation at high energies \( (s \gg m_h^2, \text{ (Higgs mass)}^2) \). The partial wave unitarity gives 14 non-linear, coupled constrains for 14 effective coupling constants depending on 5 primordial Higgs couplings and two vacuum expectation values. These can be turned over into upper bounds for Higgs masses which are not far away from the usual bound of 1 TeV for a single neutral Higgs boson, even if the bounds are reached in the interplay of several phenomena.

We conclude in Sect. 4; an appendix contains the vector boson–Higgs–Higgs couplings in the model.

2. The Model

We discuss the Weinberg–Salam model with two complex \( Y = 1 \) scalar doublets [6]

\[
\Phi_1 = \begin{pmatrix} F_1^+ \\ F_1 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} F_2^+ \\ F_2 \end{pmatrix}
\]  

(1)
having the vacuum expectation values

\[
\Phi_1 \rightarrow \left( \begin{array}{c} 0 \\ v_1 \end{array} \right), \quad \Phi_2 \rightarrow \left( \begin{array}{c} 0 \\ v_2 \end{array} \right),
\]

(2)

The Higgs potential is assumed to be

\[
V(\Phi_1, \Phi_2) = \lambda_1 (\Phi_1^+ \Phi_1 - v_1^2)^2 + \lambda_2 (\Phi_2^+ \Phi_2 - v_2^2)^2 
+ \lambda_4 \left( (\Phi_1^+ \Phi_1 - v_1^2)^2 + (\Phi_2^+ \Phi_2 - v_2^2)^2 \right)
+ \lambda_4 \left[ (\Phi_1^+ \Phi_1)(\Phi_2^+ \Phi_2) - (\Phi_1^+ \Phi_2)(\Phi_2^+ \Phi_1) \right]
\]

\[
- \frac{1}{4} \lambda_6 (\Phi_1^+ \Phi_1 - \Phi_2^+ \Phi_2)^2.
\]

(3)

Under CP-invariance and the discrete symmetry

\[
\Phi_1 \rightarrow - \Phi_1, \quad \Phi_2 \rightarrow \Phi_2,
\]

this is the most general potential. \(\lambda_1, \lambda_2\) are positive.

In order to find the couplings of the Higgs fields from (3), we must determine the Higgs fields in terms of the unphysical fields \(\Phi_1, \Phi_2\). This can be done by diagonalizing the Higgs mass matrix. In the charged sector we get for the masses of the charged Higgs fields \(H^\pm\)

\[
m_{H^\pm}^2 = \lambda_4 (v_1^2 + v_2^2) = \lambda_4 \frac{2 m^2_H}{g^2},
\]

(4)

g means the usual SU(2) coupling constant, as well as

\[
F_1 = v_1 + \frac{1}{\sqrt{2}} (\Phi^0 \cos \beta - h^0 \sin \beta)
+ \frac{i}{\sqrt{2}} (\Phi^0 \cos \beta - h^0 \sin \beta),
\]

(6)

\[
F_2 = v_2 + \frac{1}{\sqrt{2}} (\Phi^0 \sin \alpha + h^0 \cos \alpha)
+ \frac{i}{\sqrt{2}} (\Phi^0 \sin \alpha + h^0 \cos \alpha),
\]

where \(G^0\) is the neutral Goldstone boson, and the neutral physical Higgs mass

\[
m_{H^0}^2 = \lambda_6 (v_1^2 + v_2^2),
\]

\[
m_{H^+}^2 = 2(a + c + d),
\]

\[
m_{H^-}^2 = 2(a + c + d).
\]

(7)

Here

\[
\sin \alpha = -\left( \frac{1}{2} - \frac{c - a}{2d} \right)^{1/2}, \quad \cos \alpha = \left( \frac{1}{2} + \frac{c - a}{2d} \right)^{1/2},
\]

\[
a = v_1^2 (\lambda_1 + \lambda_3), \quad b = v_1 v_2 \lambda_3, \quad c = v_2^2 (\lambda_2 + \lambda_3),
\]

\[
d = (c - a)^2 + 4b^2)^{1/2}.
\]

(8)

Further details of the model are described in [11].

3. Upper Bounds

Substituting (5), (6) into (3) yields the physical Higgs self-couplings containing quadratic mass terms, trilinear and quartic Higgs couplings. Since we will discuss only tree graphs in the transitions of the type \(Higgs + Higgs \rightarrow Higgs + Higgs\) at very high energies, \(s \gg (\text{Higgs mass})^2\), we can disregard of the trilinear Higgs couplings leading to entirely negligible contributions in tree graph approximation at very large energies.

The remaining quartic couplings are as follows

\[
V_{\text{phys}} = g_1 H^+ H^- h^0 \phi^0 + \frac{1}{2} g_2 H^+ H^- h^0 H^0 + \frac{1}{2} g_4 H^+ H^- H^0 h^0 + \frac{1}{2} g_5 (H^+ H^-)^2 + \frac{1}{2} g_6 h^0 H^0 H^0 + \frac{1}{2} g_7 h^0 H^0 h^0 \phi^0 + \frac{1}{2} g_8 h^0 H^0 h^0 h^0 + \frac{1}{2} g_9 h^0 h^0 h^0 h^0 + \frac{1}{2} g_{11} H^0 H^0 H^0 + \frac{1}{2} g_{12} H^0 H^0 H^0 H^0 + \frac{1}{2} g_{13} H^0 H^0 H^0.
\]

(9)

The effective couplings \(g_i\)'s are

\[
g_1 = \frac{1}{2} \lambda_6 \sin 2(\beta - \alpha) + \sin 2(\alpha (\lambda_2 \cos^2 \beta - \lambda_1 \sin^2 \beta)),
\]

\[
g_2 = \lambda_4 \sin^2 (\beta - \alpha) + 2 \lambda_3 + 2 \lambda_2 \cos \alpha \sin \beta + 2 \lambda_1 \sin^2 \alpha \sin \beta,
\]

\[
g_3 = 2 (\lambda_3 + \lambda_2 \cos^4 \beta + \lambda_1 \sin^4 \beta),
\]

\[
g_4 = \lambda_4 \cos^2 (\beta - \alpha) + 2 \lambda_3 + 2 \lambda_2 \cos^2 \beta \sin \alpha + 2 \lambda_1 \sin^2 \beta \cos \alpha,
\]

\[
g_5 = 2 g_3,
\]

\[
g_6 = \frac{1}{2} \lambda_6 \sin 2(\beta - \alpha) + \sin 2(\alpha (\lambda_2 \cos^2 \beta - \lambda_1 \sin^2 \beta)),
\]

\[
g_7 = 3 \sin 2 \alpha (\lambda_2 \sin^2 \alpha - \lambda_1 \cos^2 \alpha),
\]

\[
g_8 = \lambda_6 \sin^2 (\beta - \alpha) + 2 \lambda_3 + 2 \lambda_2 \cos^2 \beta \sin \alpha + 2 \lambda_1 \sin^2 \beta \cos \alpha,
\]

\[
g_9 = 2 \lambda_3 + \frac{3}{2} \sin^2 2 \alpha (\lambda_1 + \lambda_2),
\]

\[
g_{10} = 3 \sin 2 \alpha (\lambda_2 \cos^2 \alpha - \lambda_1 \sin^2 \alpha),
\]

\[
g_{11} = 6 (\lambda_3 + \lambda_2 \cos^4 \alpha + \lambda_1 \sin^4 \alpha),
\]

\[
g_{12} = \lambda_6 \cos^2 (\alpha - \beta) + 2 \lambda_3 + 2 \lambda_2 \cos^2 \beta \sin \alpha + 2 \lambda_1 \sin^2 \beta \cos \alpha,
\]

\[
g_{13} = 3 g_3,
\]

\[
g_{14} = 6 (\lambda_3 + \lambda_2 \sin^4 \alpha + \lambda_1 \cos^4 \alpha).
\]

In scattering processes defined by (9) only contact graphs must be taken into account. Namely, the vector boson–Higgs–Higgs couplings (see Appendix) give vanishing, or logarithmically divergent terms (in \(t\) channel vector boson exchange graphs) at very high energies due to their derivative nature. Fortunately, this logarithmic violation of unitarity occurs only at exponentially high energies of the order

\[
m_{W,Z} \exp (\gamma \gamma), \quad \gamma = \frac{\sqrt{2} \pi}{\pi},
\]

(mixing-angle terms are \(O(1))\) so that, below this limit the vector meson exchange is small compared to the contact graphs of \(O(1))\).

The partial wave unitarity restricts a partial wave \(f_j\) by \(|f_j| < 1\). In our case zeroth order partial waves occur, thus \(|g_i| < 16 \pi, \quad i = 1, \ldots, 14\). Given \(v_1, v_2\), these inequalities impose restrictions on \(\lambda_k\). Once \(\tilde{g}_i\).