The $S_{11} - P_{11}$ Phase Shifts of Pion-Nucleon Scattering

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Abstract. The $S_{11}$ and $P_{11}$ partial amplitude is calculated using the $N/D$ method. The present calculation differs from the previous ones in the handling of the divergent behavior of the forces due to first-order diagrams, by parameterizing the short-range forces, and in the way of including inelasticity. We obtain the nucleon as a bound state with correct mass and residue and the correct $S_{11}$ and $P_{11}$ scattering lengths. The calculated $S$-wave and $P$-wave phase shifts and absorption coefficients are in substantial agreement with those from the CERN Theoretical and Almehed-Lovelace phase shift analyses.

I. Introduction

There has been considerable interest in the $S_{11} - P_{11}$ amplitude of $\pi N$ scattering in the last twelve years [1-10]. One would like to calculate this amplitude by means of a single-channel $N/D$ calculation in which the input long-range forces are given by the usual $N$, $\rho$, and $N_{33}$ first-order exchange diagrams. The calculated amplitudes should reproduce both the $S$ and $P$-wave phase shifts and absorption coefficients and, in particular, their scattering lengths. Moreover, since the $P_{11}$ amplitude has a pole corresponding to the exchange of the nucleon in the direct channel, an important question to be considered here is whether this pole corresponds to a bound state or to an elementary particle. In the $N/D$ formalism a pole in the amplitude which is caused by a zero in $D$ is regarded as a bound state. If the pole has to be inserted as a pole in $N$, then it is regarded as an elementary particle. Coulter and Shaw [4] did obtain the phase shifts correctly but could not obtain at the same time a bound state. However, they cautioned from taking their calculations too seriously and concluding from them the elementarity of the nucleon. Some authors [1, 2, 4, 10] have succeeded in obtaining the bound state but without getting even the correct sign for the $S$-wave scattering length. Thus the fundamental question of whether the nucleon is composite or elementary has not been settled in the literature. One might think that the long-range forces are not good enough [4, 10], but these forces are perfectly adequate for reproducing all other pion-nucleon partial waves [4]. The single-channel $N/D$ method was thought [10] to be inadequate for calculating this intriguing partial amplitude. In this work we assume, as usual, that the long-range forces are given by the first-order exchange graphs of the $\rho$, $N$ and $N_{33}$ particles. The "generalized potentials" due to $\rho$ and $N_{33}$ exchange behave like $W^0$ and $W^1$ for large energies and make it impossible to solve the $N/D$ equation. Therefore one usually introduces a cut-off [4, 10] to the dispersion integral of the integral equation for the $N$ function. This procedure does not have any justification. Instead, we deal with the above difficulty by decomposing the forces (or the generalized potentials) due to these diagrams into long-range and short-range forces, according to whether the corresponding cuts are
near or far from the physical region. We then approximate all short-range forces, including those contributed by higher order diagrams, by a pair of complex-conjugate poles on the imaginary axis of the $W$-plane whose position and complex residue are parameters. The nearby cuts lead to contributions which vanish as $W^{-1}$ at infinity. We adopt a method for incorporating the inelasticity originated by Chew and Mandelstam [11] and developed by Rothleitner and Stech [12], instead of the more commonly used method of Frye and Warnock [13]. Furthermore, since the $P$-wave phase shift vanishes near the inelastic threshold, the $P$-wave inelasticity function $R = \frac{\sigma_{inel}}{\sigma_{el}}$ is assumed to have a resonant shape in that neighborhood which depends on a parameter (see Sect. IV). $R$ for both the $S$-wave and the $P$-wave is assumed constant for energies beyond the experimental limit and these two constants constitute two additional parameters bringing the total to six. By varying these parameters we are able to obtain the nucleon as a bound state with correct mass and residue along with values for the $S$- and $P$-wave scattering lengths and $S$- and $P$-wave phase shifts and absorption coefficients which agree substantially with experiment.

II. Notation and Kinematics

In the following we use the usual notations [2]. $W$ and $q$ are the center of mass energy and three-momentum, $M$ and $\mu$ are the masses of the nucleon and pion, $E$ is the c.m. energy of the nucleon. We use units in which $\hbar = c = 1$.

Instead of the usual partial wave amplitudes $f_{1\pm}(W)$, we use the amplitudes

$$ h_{1\pm}(W) = \frac{f_{1\pm}(W)}{\rho_{1\pm}(W)} q(W) \quad (2.1) $$

which are related to the physical phase shifts $\delta_{1\pm}$ and absorption coefficients $\eta_{1\pm}$ by

$$ h_{1\pm}(W+i\epsilon) = \frac{n_{1\pm}(W)e^{2i\delta_{1\pm}(W)} - 1}{2i\rho_{1\pm}(W+i\epsilon)} \quad \text{for } W > W_0 \quad (2.2) $$

where the kinematical factors $\rho_{1\pm}(W)$ are introduced to remove the kinematic double pole of $f_{1\pm}(W)$ at the origin. In particular, we define

$$ \rho_{1-}(W) = \frac{E-M}{2W} q = \frac{q^3}{(W+M)^2 - \mu^2} \quad \frac{w-w_3}{4M(M+\mu)} \quad (2.3) $$

and

$$ \rho_{0+}(W) = \frac{E+M}{2W} q = \frac{q^3}{(W-M)^2 - \mu^2} \frac{w-w_4}{4M(M+\mu)} \quad (2.4) $$

where $W_0 \equiv M+\mu$ is the threshold energy. The analytic continuation of $\rho_{1-}(W)$ in the left half of the $W$-plane is related to $\rho_{0+}(W)$ by

$$ \rho_{1-}(W) = \rho_{0+}(-W), \quad (2.5) $$

provided we stay on the first sheet, where $q(-W) = q(W)$, i.e. provided the continuation path does not cross the short cut $-M+\mu \leq W \leq M-\mu$ of the function $q(W)$ in the $W$-plane. Eq. (2.5) and the usual MacDowell symmetry for $f_{1\pm}(W)$ give the corresponding relation for the $h_{1\pm}(W)$

$$ h(W) \equiv h_{1-}(W) = -h_{0+}(-W) \quad (2.6) $$

on the first sheet. From now on we shall use $h(W)$ instead of $h_{1-}(W)$ for simplicity.

From the usual expressions for the $f_{1\pm}(W)$ in terms of the invariant amplitudes we obtain

$$ h_{1-}(W) = \frac{1}{4\pi} \left[ \frac{E+M}{E-M} (A_1 - MB) \right. $$

$$ - (A_0 - MB) + W \left( B_0 + \frac{E+M}{E-M} B_1 \right) \left] \quad (2.7) $$

$$ h_{0+}(W) = -\frac{1}{4\pi} \left[ \frac{E-M}{E+M} (A_1 - MB) \right. $$

$$ - (A_0 - MB) - W \left( B_0 + \frac{E-M}{E+M} B_1 \right) \left] \quad (2.8) $$

which also immediately leads to Eq. (2.6), as expected.

Here

$$ A_1(s) = \frac{1}{2} \int_{-1}^{1} A^\pm(s, t) P_l(x) dx, \quad x = \cos \theta \quad (2.9) $$

and similarly for $B_1$. $A^\pm(s, t)$ and $B^\pm(s, t)$ are the usual invariant amplitudes of isospin $\frac{1}{2}$.

III. The Input Forces

We assume that to a good approximation only the first order exchange graphs of $\rho$, $N$ and $N_3$ describe the long-range forces. These graphs give rise to the following invariant amplitudes [4,14]

$$ \frac{1}{4\pi} A^\pm(s, t) = -4g_{33} \left[ a - (m_{33} + M)s \right. $$

$$ - 3C_2 \left. \frac{2s - \Sigma + m_p^2}{t - m_p^2} \right], \quad (3.1) $$

and similarly for $B^\pm$.

$$ \frac{1}{4\pi} B^\pm(s, t) = -4g_{33} \left[ b - \right.$$

$$ \left. \frac{2s - \Sigma + m_p^2}{u - m_p^2} - 3C_2 \left. \frac{2s - \Sigma + m_p^2}{t - m_p^2} \right] $$

$$ + \left. \frac{6(C_1 + 2MC_2)}{t - m_p^2} \right]. \quad (3.2) $$