Inelastic Proton Amplitudes for $f$-Wave Resonances in $^{55}$Co

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Received October 3, 1979

The measurement of inelastic proton channel amplitudes has been extended to $f$-wave resonances. Ten resonances were studied in the $^{54}$Fe($p,p'\gamma$) reaction. All were established to have $J^\pi = 5/2^-$, and inelastic decay amplitudes were determined for these resonances. Elastic and inelastic spectroscopic factors were determined for the $5/2^-$ analogue state at $E_p = 3.80$ MeV.

1. Introduction

In recent years, a method [1, 2] for determining the magnitudes and relative phases of inelastic decay amplitudes for isolated resonances has been developed and applied to several related problems. When combined with very good beam energy resolution, this technique permits a rapid study of a large number of resonances. The method was recently applied [3, 4] to a fragmented analogue state in $^{45}$Sc, where non-statistical behavior of the relative sign between the inelastic amplitudes of the analogue fine structure resonances was demonstrated, and other aspects of analogue state theory were successfully tested. Future experiments will utilize the increased sensitivity of this technique to investigate the statistical behavior of reduced width amplitudes in analogue-free regions. The present experiment extends the method to $f$-wave resonances and to four inelastic channels; previous measurements at TUNL were restricted to $p$-wave resonances and two inelastic channels. The sensitivity of these measurements to small admixtures of higher $l'$ values is demonstrated for an $f_{5/2}$ analogue in $^{55}$Co. This analogue, which had been investigated with high resolution at this laboratory [5], was chosen for study in part because measurements [6] with the ($^3$He, $dp$) reaction indicated a preferred solution with 23% of the total inelastic width in the $l' = 3$ channel. Resonances studied in the present work included the five fragments of the $f$-wave analogue at $E_p = 3.80$ MeV, as well as five other $f$-wave resonances well removed from the analogue. Spectroscopic applicability is demonstrated by determining the $p_{1/2}$ and $p_{3/2}$ amplitudes, and the $l' = 3$ admixture for this analogue. As is described below, the spins of all 10 resonances are confirmed to be 5/2, and the $l' = 3$ admixture in the analogue is shown to be about 3%.

A short derivation of the angular distribution equations, and a discussion of mixing ratios for fragmented analogue states are given in the next section. The experimental procedure and the data processing and analysis are described in Sect. 3. The results of this analysis are presented and discussed in Sect. 4, while the final section contains a summary of these results.

2. Theory

2.1. Angular Distribution Equations

The use of angular distributions in the ($p,p'\gamma$) reaction to obtain inelastic channel amplitudes and...
relative phases was described for $p_{3/2}$ resonances by Dittrich et al. [2]. More general angular distribution expressions for this reaction are given by Kraus et al. [7]. In the present work, the following specializations are made: even-even target nuclei are assumed, giving a ground state spin and parity of $0^+$. Inelastic scattering proceeds through an isolated compound nuclear resonance ($J=j$, $\pi=\pi$) to an excited $2^+$ state, which decays by a quadrupole gamma-ray transition to the $0^+$ ground state. These assumptions lead to, in the total angular momentum representation, the following angular distributions:

\[
W(\theta) \propto \sum (-1)^{l(l-j)} Z(l;j_1)Z(l';j_2;j_1;j_2k) W(\theta) \propto \sum (-1)^{l(l-j)} Z(l;j_1)Z(l';j_2;j_1;j_2k)
\]

where $P_k$ is the $k$-th Legendre Polynomial, and the quantities in brackets $\langle \rangle$ are the nuclear matrix elements. Parity conservation requires $l+l'=\text{even}$ number, and the coupling coefficients further restrict each sum to only a few terms. Moreover, Coulomb penetrabilities limit the $l'$ values needed to describe the data to the lowest allowed values. In previous work with $p$-wave resonances, only $l'=1$ was included in the analysis, since the $l'=3$ penetrability in this mass and energy region is about 2% of the $l'=1$ penetrability. For inelastic scattering from $f$-wave resonances, $l'=1$, 3 and 5 are allowed. Although, again, the $l'=3$ penetrability is only about 2% of the $l'=1$ value, Fortier et al., in a $^{54}$Fe($^3$He, dp) experiment [6], observed an $l'=3$ admixture of more than 20% for the $f$-wave analogue in question. In the present analysis, both $l'=1$ and $l'=3$ contributions have been included. Keeping the first two terms in the sums over $l'$ ($l'_u$ for the lower value and $l'_v$ for the upper) results in angular distributions which depend on at most four amplitudes: $\langle l'=l'_u+1/2 \rangle$ and $\langle l'=l'_u-1/2 \rangle$. For convenience in describing the relative angular distributions, the following mixing ratios are defined:

\[
\delta_L = \frac{\langle l'=l'_u+1/2 \rangle}{\langle l'=l'_u-1/2 \rangle} \quad \delta_U = \frac{\langle l'=l'_u+1/2 \rangle}{\langle l'=l'_u-1/2 \rangle}
\]

and the corresponding mixing angles:

\[
\tan (\phi_L) = \delta_L \quad |\phi_L| \leq 90^\circ
\]

\[
\tan (\phi_U) = \delta_U \quad |\phi_U| \leq 180^\circ
\]

Restricting $|\phi_L|$ to $90^\circ$ is equivalent to arbitrarily choosing the amplitude with the lowest $j'$ value to have positive sign ($\langle j' \text{ min} \rangle \propto \cos (\phi_L)$) and measuring the signs of the other amplitudes relative to this one. There remains one more independent variable, chosen to describe the $l'=l'_u$ strength relative to the total strength:

\[
\varepsilon^2 = \frac{|\langle l'=l'_u \rangle|^2}{|\langle l'=l'_u \rangle|^2 + |\langle l'=l'_v \rangle|^2}
\]

In order to work in terms of reduced width amplitudes instead of matrix elements, it is necessary to take into account a phase shift which depends on $l'$. All other effects, such as energy dependence, cancel when ratios of matrix elements are taken. The net result is a factor of $\cos (\alpha_u - \alpha_v)$ which multiplies the $l'$ interference terms (see, for example, the review article by Biedenharn [8]), where $\alpha_v$ includes both the Coulomb phase shift and the hard sphere phase shift for the $l'$ partial wave. The size of this correction factor is usually small for $f$- and $p$-wave resonances in the mass and energy region studied at this laboratory. Therefore the effect of the higher $l'$ amplitudes goes as $\varepsilon^2$ and not as $|\varepsilon|$ as would otherwise be the case. For $d$-wave resonances the cosine factor is approximately $-0.5$ and the interference term does have a significant effect. Substituting Eqs. (2)–(4) into Eq. (1) yields equations of the form

\[
W(\theta) \propto 1 + \sum a_k P_k(\theta) \quad k = 2, 4, \ldots
\]

where the $a$'s are functions of $\phi_L$, $\phi_U$, and $\varepsilon^2$. For $p_{3/2}$ resonances the only non-zero terms are $a_{2 \nu}$ and $a_{2 \sigma}$. Therefore one makes the assumption $\varepsilon^2 = 0$ and solves for $\phi_U$. Consistency in the solutions of the two equations for $\phi_L$ ($a_{2 \nu}$ and $a_{2 \sigma}$) indicates that the effects of $l'=3$ are small for the $p$-wave resonances measured thus far [9]. This is to be expected since $\cos (\alpha_1 - \alpha_3)$ is small. For $f_{3/2}$ resonances, the angular distribution coefficients are:

\[
a_{2 \nu} = (1 - \varepsilon^2) \left( \frac{16}{5} \sqrt{\frac{2}{7}} \sin \phi_L \cos \phi_L - \frac{4}{35} \sin^2 \phi_L \right)
\]

\[
+ \left( \varepsilon^2 (1 - \varepsilon^2) \right)^{1/2} \left( -\frac{12}{5} \sqrt{\frac{2}{7}} \cos \phi_L \cos \phi_U + \frac{192}{245} \sin \phi_L \cos \phi_U 
\right.
\]

\[
- \frac{108}{245} \sin \phi_L \sin \phi_U \cos (\alpha_1 - \alpha_3)
\]

\[
+ \varepsilon^2 \left( \frac{4}{35} \cos^2 \phi_U + \frac{8}{35} \sin \phi_U \cos \phi_U + \frac{10}{21} \sin^2 \phi_U \right),
\]