The $^{142}_{\text{Ce}}(d, p)^{143}_{\text{Ce}}$ Reaction and a Quasi-$f_{7/2}$ Pattern in $^{143}_{\text{Ce}}$ in the Cluster-Vibration Model

G. Vanden Berghe  
Seminarie voor Wiskundige Natuurkunde, Rijksuniversiteit Gent, Gent, Belgium

V. Paar  
Prirodoslovno-matematički fakultet, University of Zagreb, Zagreb and "Rudjer Bošković" Institute, Zagreb, Yugoslavia

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In the framework of the cluster-vibration model we discuss a quasi-$f_{7/2}$ pattern of low-lying states in $^{143}_{\text{Ce}}$ with the $I = j - 2$ anomaly, and its consequences on the spectroscopic factors for this nucleus.

1. Introduction

In the framework of the cluster-vibration model (CVM), $N=85$ odd-$A$ nuclei are described by three particles in the $N=82-126$ shell (cluster) coupled to the quanta of the quadrupole vibration [1-6]. In the $N=82-126$ shell, the $f_{7/2}$ state is the lowest single-particle state, rather separated from the other single-particle states in the same shell. $N=85$ nuclei, therefore, partly resemble $Z=23$ or $N=23$ nuclei, the so-called $f_{7/2}$ nuclei [7], and will be referred to as quasi-$f_{7/2}$ nuclei. In Refs. 3, 6, $f_{7/2}$ nuclei were treated in the cluster-vibration model. In the systematics of $N=85$ nuclei [8-19], the triplet of states $3/2^-$, $5/2^-$, $7/2^-$ appears as a lowest-lying group of states. The $3/2^-$ member of the triplet is the highest one in $^{149}_{\text{Gd}}$ (352 keV above the $7/2^-$ ground state) and thus reflects the approximate shell-closure at $Z=64$. This state is systematically lowered in passing towards lighter $N=85$ nuclei. In $^{147}_{\text{Sm}}$, this state lies 197 keV and in $^{145}_{\text{Nd}}$, it lies 67 keV above the $7/2^-$ ground state. In $^{143}_{\text{Ce}}$, the $3/2^-$ state is the ground state, lowered by 19 keV below the $7/2^-$ state. This effect is referred to as the $I = j - 2$ anomaly and can naturally be accounted for in $f_{7/2}$ nuclei and quasi-$f_{7/2}$ nuclei as a consequence of the cluster-vibration coupling [6]. Both the $3/2^-$ state and the $5/2^-$ state are predicted to be collective and to have small spectroscopic factors for $(d, p)$ transfer reactions. In the present paper we discuss the spectroscopic factors and the quasi-$f_{7/2}$ pattern with the $I = j - 2$ anomaly in $^{143}_{\text{Ce}}$ in the cluster-vibration model.

The Hamiltonian of the cluster-vibration model in its simplest version is

\[ H = H_{\text{SM}}^{(n)} + H_{\text{VIB}} + H_{\text{PV}}^{(n)} + H_{\text{P}}^{(n)}. \]  

Here $H_{\text{SM}}^{(n)}$ describes $n$ valence-shell particles or holes (cluster) in spherical shell-model potential; $H_{\text{VIB}}$ represents the free quadrupole vibrational field (phonons). $H_{\text{PV}}^{(n)}$ is the particle-vibration interaction between the $n$ valence-shell particles and phonons. The particle-vibration coupling strength will be denoted by $a$. $H_{\text{P}}^{(n)}$ is the pairing residual force acting between the $n$ single particles of the cluster. The Hamiltonian (1) is diagonalized in the basis built from vectors antisymmetrized in $n$ valence-shell particles and symmetrized in quadrupole phonons:

\[ \langle j_1 \ldots j_n | J, NR; I \rangle. \]

Here $J$ denotes the total angular momentum of the single-particle states of the cluster; $N$ is the number of quadrupole phonons of angular momentum $R$. The cluster angular momentum $J$ and the phonon angular momentum $R$ are coupled to the total angular momentum $I$ of the basis state. The cluster contains respectively three and two single particles in $^{143}_{\text{Ce}}$ and in $^{142}_{\text{Ce}}$.

The cluster-vibration model does not introduce new parameters with respect to the particle-vibration model of Bohr and Mottelson [19] or with respect to the quasi-particle-vibration model of Kisslinger and Sorensen [20]. An essential novelty of the CVM is the explicit inclusion of the Pauli principle in the valence shell. In this way the following important physical correlations are included:
(i) the explicit appearance of broken and promoted pairs,
(ii) the anharmonic structure of the neighbouring even-even nuclei.

The effects of these correlations, combined with the particle-vibration coupling, are significant, and the description and understanding of experimental data is thus appreciably improved [1-6].

2. The Quasi-f7/2 Spectrum of 143Ce in the Cluster-Vibration Model

In calculating the spectrum and wave functions we used the following parametrization (hereafter referred to as calculation (a)). We used the single-particle positions as centres of gravity determined from (d, p) reactions: \( e(f_{7/2})=0 \), \( e(p_{1/2})=1.94 \), \( e(p_{3/2})=1.12 \), \( e(f_{5/2})=1.78 \), \( e(h_{9/2})=1.21 \) and \( e(i_{13/2})=1.69 \) MeV [8]. The phonon energy was \( \hbar \omega=1 \) MeV, adopted as an average phonon energy for medium-heavy nuclei [5, 6]. The pairing strength was \( G = 22/A = 0.15 \) MeV.

The basis space was truncated so that the maximum dimension of the energy matrix was smaller than 155. The particle-vibration coupling strength was treated as an adjustable parameter; it was chosen to be \( a = 0.9 \) MeV in order to obtain overall agreement with the experimental spectrum. This value is very close to the particle-vibration coupling strength used for the cluster-vibration-model calculation [3] for \( N=53 \) nuclei (\( a = 0.8 \) MeV).

Both experimentally and theoretically, we observe a particular grouping of the low-lying states: the triplet \( (3/2^-, 7/2^-, 5/2^-) \) is followed by the quadruplet \( (11/2^-, 9/2^-, 3/2^-, 1/2^-) \) and this is followed by the next group of states.

In the zeroth-order approximation (\( a = 0 \) MeV), the lowest-lying \( (f_{7/2}^2)J/2 \) cluster of seniority one, lowered by the pairing force, is followed by clusters of seniority three

\[
| (f_{7/2}^3) J = 3/2, 5/2, 9/2, 11/2 \rangle
\]  

and by the one-phonon multiplet

\[
| (f_{7/2}^2) J = 7/2, 11/2 \rangle \]  

which is based on the lowest-lying cluster. By the inclusion of the particle-vibration coupling strength, the cluster states (2) mix strongly with the corresponding multiplet states (3). As a consequence, the low-lying triplet \( 3/2^- \), \( 7/2^- \), \( 5/2^- \) is generated. The influence of the Pauli principle, reflected in the corresponding matrix elements between the three-particle configurations and amplified by the effect of the particle-vibration coupling, leads systematically to the lowering of the \( 5/2^- \) state with respect to the other low-lying states. On the other hand, the \( 3/2^- \) state is strongly influenced by the presence of the single-particle state of angular momentum \( I = J = 2 = p_{3/2} \), producing an appreciable shift downwards [6]. For a sufficiently strong but still intermediate particle-vibration coupling strength, this state crosses the \( I = J \) (i.e. \( 7/2^- \)) state and the \( I = J - 1 \) (i.e. \( 5/2^- \)) state and becomes the ground state. This is the so-called \( I = J - 2 \) anomaly. In addition, all three states of the triplet are of a strong collective character. This is a well-known feature of some \( N=23 \) and \( Z=23 \) nuclei, and has been extensively studied within the framework of the CVM [6]. Because of the appearance of the same pattern in \( N=85 \) nuclei, we classify these nuclei as quasi-\( f_{7/2} \) nuclei. The analogy can be pursued even further, because the collective \( 11/2^- \) and \( 9/2^- \) states appearing in the quadruplet of states above the lowest-lying triplet, appear both in \( f_{7/2} \) and in \( N=85 \) nuclei. The most pronounced components in the corresponding wave functions are again (2) and (3). The remaining two states of the quadruplet in \( N=85 \) nuclei are largely due to clusters and multiplets involving \( f_{7/2}^2 p_{3/2} \) configurations. These two states, therefore, appear as intruder states characteristic of quasi-\( f_{7/2} \) nuclei, without its counterpart in \( f_{7/2} \) nuclei, in which the \( p_{3/2} \) single-particle configuration lies much higher than in \( N=85 \) nuclei (a few MeV above the \( f_{7/2} \) position).

3. Spectroscopic Factors of Quasi-f7/2 States

In this section we discuss the spectroscopic factors for the \( ^{142}\text{Ce}(d, p)^{143}\text{Ce} \) transfer reaction [8] within the cluster-vibration model [2, 4]. The \( ^{142}\text{Ce}_{84} \) and \( ^{142}\text{Ce}_{85} \) nuclei are described by coupling two- and three-particle clusters to quadrupole vibrations, respectively. By diagonalizing the Hamiltonian matrix in the same parametrization, we obtain the wave function for the target ground state, \( ^{142}\text{Ce} \):

\[
|0^+\rangle = \sum_{j_1, j_2, JNR} C^{(0)}_{(j_1, j_2, J), NR} |(j_1, j_2) J, NR \rangle |0\rangle
\]

and the wave functions of the states in the daughter nucleus, \( ^{143}\text{Ce} \):

\[
|I^-\rangle = \sum_{j_1, j_2, J_1, J_2, JNR} C^{(I^-)}_{(j_1, j_2, J_1, J_2), J, NR} |(j_1, j_2) J_1, J_2, J, NR \rangle |I^-\rangle
\]

Inserting (4) and (5) into the expression for the spectroscopic factor

\[
S_{ij} = \langle IM | [\Phi_i \otimes 00] \rangle |0\rangle^2.
\]