Abstract. It is shown that the $D^0 - ar{D}^0$ mixing is strongly suppressed in the Standard Model and is smaller by several orders of magnitude than the current experimental upper-bound. The detection of a stronger $D^0 - ar{D}^0$ mixing in the future experiments might indicate the existence of a new physics beyond the Standard Model. As an example, it is shown that in a class of simple super-symmetric theories $D^0 - ar{D}^0$ mixing could be substantially larger than the Standard Model prediction. If on the other hand the Standard Model prediction is confirmed by future experiments that will require that the super-symmetric extension of the standard model should have a much larger SUSY breaking scale than that indicated by the phenomenology of the neutral kaon system.

1. Introduction

The computation of the $K_L - K_S$ mass difference ($\Delta m_K$) in gauge theories is of considerable interest. Historically this was first done by Gaiard and Lee [1] in the standard model [2] with four quarks and the charmed quark mass was estimated. This calculation was subsequently generalised to a six quark model by Inami and Lim [3]. Since then the theoretical estimates of $\Delta m_K$ and other experimentally accessible parameters of the $K^0 - \bar{K}^0$ system (e.g., the CP violation parameter) have been used to predict the values of the Kobayashi-Maskawa angles [4], the upper bound on the top quark mass [5] etc.

In all the above calculations the relevant box diagrams were calculated in the zero external momentum approximation (i.e., by neglecting the three momenta and the masses of the quarks on the external lines). This approximation, though somewhat justified for $K^0 - \bar{K}^0$ mixing, is unreliable for the mixing of heavy pseudo-scalar mesons ($D^0 - \bar{D}^0$, $B^0 - \bar{B}^0$ etc.). A different approximation scheme is, therefore, called for. This was done in a four-quark model by Datta and Niyogi [6]. Subsequently Hagelin discussed $B^0 - ar{B}^0$ mixing in a six quark model [7] using a different approximation. This approximation involving the neglect of all four-momenta appearing in gauge boson propagators, however, appears to be unreliable because it does not reproduce quite a few important contributions from the box diagram. This is shown in the next section by using the techniques used in [6].

We have generalized the results of [6] to a six quark model. In [6] only the approximate formulas for the box diagram with heavy external legs were given*, but no attempt was made to calculate physically interesting parameters of the $D^0 - \bar{D}^0$ system. Using the results of our analysis of the box diagram in the six quark model, we have calculated the usual parameters $r$, $\bar{r}$ and $a$ (defined below) of the $D^0 - \bar{D}^0$ system. Our computation reveals that all these parameters are much smaller than claimed in a recent work [8] which indicated that $D^0 - \bar{D}^0$ mixing was almost of the same order or magnitude as $K^0 - \bar{K}^0$. We find that in the standard model $D^0 - \bar{D}^0$ mixing should be almost unobservable and is predicted to be several orders or magnitude smaller than the present experimental upper bound (to be discussed below). There is, therefore, the interesting possibility that future experiments would indicate departure from the standard model if $D^0 - \bar{D}^0$ mixing turns out to be significantly larger than the standard model pre-

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* Though the basic equations (2a)-(2b) of [6] are correct, the simplified formulas (6b), (6c) and (11a) involve several algebraic errors in computation.
diction. As an example we have discussed (qualitatively) $D^0\!-\!\bar{D}^0$ mixing in super-symmetric theories to indicate that it could be much larger in such theories.

In Sect. 2 we have reviewed the approximation schemes for computing the box diagram when external momenta are not negligible and have also compared the results of [6] and [7]. In Sect. 3 we have computed the physically interesting parameters of the $D^0\!-\!\bar{D}^0$ system. Our discussions and conclusions are given in Sect. 4.

2. The Box Diagram

A typical box diagram contribution to the mixing of pseudo-scalar mesons is given in Fig. 1. The amplitude involves integrals which are hard to evaluate unless all external momenta are dropped. However, as emphasized earlier, this approximation is not justified for $D^0\!-\!\bar{D}^0$ mixing. The computation of the integrals with the external momenta retained is relatively easier if one uses the standard approximation used when one parameter in the amplitude is much larger than other relevant quantities [9]**. In our case the obvious choice for the above large parameter is the gauge boson mass ($M_w$)**. Since in this section our main purpose is to establish a valid approximation scheme for evaluating the box diagram, we restrict ourselves to a four quark model calculation. A six quark model calculation of $D^0\!-\!\bar{D}^0$ mixing is a straightforward generalization and will be discussed in the next section. The amplitude of Fig. 1 in a four quark model is given by [6]

$$A = g^4 \sin^2 \theta_c \cos^2 \theta_c J_{\mu}(-p_2, -p_4) J_{\nu}(p_3, p_1) I_{\mu\nu},$$

where

$$J_{\mu}(p_1, p_2) = \bar{q}(p_1) J_\mu(1 - \gamma_5) q'(p_2),$$

$$I_{\mu\nu} = \frac{i}{32\pi^3 M_w^2} \left[ \delta_{\mu\nu} I_1 + (p_1 + p_2)_\mu(p_1 + p_2)_\nu I_2 \right].$$

$\theta_c$ is the Cabibbo angle, $g$ is the gauge coupling constant, $q$ and $q'$ stand for spinors accompanying the destruction of $u$ and $c$ quarks ($s$ and $d$ quarks) for $D^0\!-\!\bar{D}^0$ mixing and

$$I_{\mu\nu} = \frac{d^4 k}{(2\pi)^4} \left[ k_\mu(p_1 + p_2 + k)^\nu \right]$$

$$\cdot \left( (p_2 + k)^2 - M_w^2 \right)^{-1} \left( (p_4 + k)^2 - M_w^2 \right)^{-1}$$

$$\cdot \left( (p_1 + p_2 + k)^2 - m_f^2 \right)^{-1} \left( (k^2 - m_f^2) \right)^{-1}$$

$$\cdot (m_f \leftrightarrow m_i) - (m_i \rightarrow m_f) - (m_f \rightarrow m_i),$$

$m_i$ and $m_f$ being the masses of the internal quark lines. Using the methods of [9] (see Appendix A), it is easy to see that the leading contributions of all the terms in (2.2) are $\sim 1/M_w^2$, i.e., independent of the quark masses. The leading contributions, therefore, cancel one another. It was first pointed out in [1] that because of this "GIM" cancellation the amplitude for $K^0\!-\!\bar{K}^0$ mixing is not of the order $G_F a$, but is suppressed by a factor $\sim m_t^2/M_w^2$. The latter comes from the first non-trivial term ($\sim m_t^2/M_w^2$) which survives after the subtractions shown in (2.2). It should be emphasized here that when all the external momenta are set equal to zero the suppression factor is of the form $m_i^2/M_w^2$ (where $m_i$ is the mass of the heaviest internal quark with $m_i \ll M_w$). It will emerge from the following discussions that a different suppression factor is involved when the external quark masses are important compared to internal quark masses which is the case for $D^0\!-\!\bar{D}^0$ mixing.

In order to estimate the suppression factor for $D^0\!-\!\bar{D}^0$ mixing, we consider a pair of terms with opposite signs in (2.2), carry out the subtraction and rederive the results of [6] after taking the large $M_w$ limit (see Appendix B for details):

$$I_{\mu\nu} = \frac{1}{32\pi^3 M_w^2} \left[ \delta_{\mu\nu} I_1 + (p_1 + p_2)_\mu(p_1 + p_2)_\nu I_2 \right].$$

$$I_1 = (a_j - a_i) \int \frac{dz}{6} \left[ 2 \ln [A z^2 + (a_j - a_i - A)z + a_i] \right.$$

$$+ \left. \frac{A}{(a_j - a_i)(z^2 - z) + (a_i - a_j)} \right] (a_j - a_i - A) + (a_i \leftrightarrow a_j).$$

$$I_2 = \frac{2(a_j - a_i)}{M_w^2} \int \frac{dz}{6} \left[ \frac{z(z^2 - z)}{(a_j - a_i)} \right.$$

$$\cdot \ln \left[ \frac{A z^2 + (a_j - a_i - A)z + a_i}{A z^2 - A z + a_i} \right] (a_j - a_i - A) + \left( a_i \leftrightarrow a_j \right).$$

where $a_i = m_i^2/M_w^2$, $a_j = m_j^2/M_w^2$, $A = (m_q + m_s)^2/M_w^2$. We have also neglected the third momenta of the external quarks compared to their masses.

In the approximation scheme of [7], on the other hand, one neglects the four momenta in the gauge...