Application of the Incoming Wave Boundary Condition to $^{16}$O+$^{16}$O and $^{12}$C+$^{12}$C Elastic Scattering

R. Wolf and U. Mosel
Institut für Theoretische Physik, Universität Gießen, Gießen, West Germany

Steven C. Pieper
Argonne National Laboratory, Argonne, Illinois, USA

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The elastic scattering of $^{12}$C+$^{12}$C and $^{16}$O+$^{16}$O has been studied in the framework of an incoming wave boundary condition model. Different logarithmic derivatives for the incoming waves have been tested and their effects investigated with the help of Fourier analyses. It is shown that a logarithmic derivative obtained from a JWKB approximation leads to strong absorption of the low partial waves while a logarithmic derivative constant for all partial waves causes reflections. These reflections are necessary to describe the high energy elastic scattering of $^{12}$C+$^{12}$C. The fits thus obtained with shallow real potentials are comparable to those obtained with deep folding potentials. It is shown that not the lowest partial waves, but those within a window just below the grazing angular momentum are most important for the higher energy $^{12}$C+$^{12}$C angular distributions.

1. Introduction

The elastic scattering of $^{16}$O+$^{16}$O and $^{12}$C+$^{12}$C has been studied very extensively over large angular and energy ranges [1-10]. Normal optical model analyses fail to fit these data satisfactorily [10-12]. Especially at high energies the cross section at 90° is underestimated if the parameters are determined to fit the cross section at forward angles. For example, it was impossible to fit the $^{12}$C+$^{12}$C angular distribution at $E_{CM}=51$ MeV (which has a minimum at 90°) with an optical model calculation using Woods-Saxon potentials [12]. Wieland et al. [12] succeeded in getting a good fit for the high energy data by using a deep folding potential (about 400 MeV deep). This is in contrast to the usual point of view that shallow potentials must be used [4, 6, 13, 14] and that the scattering predictions are not sensitive to the interior region of the potential.

The insensitivity of earlier calculations to the interior region [4, 15] lead to the incoming wave boundary condition (IWBC) model. The main idea was to describe strong absorption by allowing only incoming waves at some certain radius, the boundary radius, while the normal optical model is adopted in the surface region. Such a strong absorption may be impossible to achieve in the normal optical model since simply increasing the depth of the imaginary potential will eventually result in increased reflection [16]. The presence of a very strong absorption at small distances is also motivated by the success of the critical distance models [17, 18] for the description of fusion. These models assume an infinitely strong friction inside a distance that approximately coincides with the boundary radius of the IWBC. Since the wave function in the overlap region must be calculated numerically, one cannot exactly divide it up into outgoing and incoming waves. The problem with the IWBC, therefore, is to get a good expression for the incoming waves. These have so far been approxi-
It is the primary aim of this paper to investigate the effects of these two methods. Special emphasis will be put on the question whether the “incoming wave boundary condition” actually lives up to its name. This point will be discussed in the next section. There it will be shown that the JWKB method indeed achieves strong absorption (but does not fit the data discussed below). The IWBC with a simplified wave number - on the other hand - creates strong reflections.

Reflections of the low partial waves have recently been shown to play a decisive role in the elastic scattering of heavy ions, like $^{12}$C+$^{12}$C (Ref. 28) and presumably also $^{16}$O+$^{16}$O. In optical model analyses of the former system these reflections were generated by the use of a deep folding potential. The authors of Ref. 33 indeed report that they are able to determine the potential down to distances of 4 fm and that they are sensitive down to 2 fm. The question that remains is then whether this potential has physical meaning in itself as a true interaction potential or whether it is only a tool to generate the reflections that are possibly due to a completely different mechanism. Possibilities for such a mechanism include sudden onset of absorption at a “critical distance” (Ref. 18) or strong (repulsive) polarizations of the ions at the contact point.

In order to help to clarify this point we have analyzed the elastic scattering data for $^{12}$C+$^{16}$C and $^{16}$O + $^{16}$O over the full range of angles and energies with an IWBC. We use here the boundary condition as a tool to generate reflections, independent of the potential depth so that these two aspects can clearly be separated from each other. These studies are reported in Sects. 3 and 4. The effects of the low partial waves are discussed in Sect. 5 whereas attempts to calculate the fusion cross section are reported in Sect. 6.

2. The Effect of an IWBC

Two kinds of incoming waves have been used so far. One was obtained from a JWKB approximation and has the form [19]

$$\chi_i(r)=\frac{1}{\sqrt{k_i(r)}} e^{-\frac{1}{2} \int k_i(r)dr} \quad (2.1)$$

where, as usual, the radial wave function is given by

$$R_i(r)=\frac{\chi_i(r)}{r} \quad (2.2)$$

Here $k_i(r)$ is the local wave number:

$$k_i(r)=\sqrt{\frac{2\mu}{\hbar^2} \left(E-V_c-\frac{\hbar^2 l(l+1)}{2\mu r^2}+V_0 f(r)+i W_0 g(r)\right)} \quad (2.3)$$

where $f(r)$ and $g(r)$ are Woods-Saxon formfactors.

From this one calculates the following logarithmic derivative

$$\frac{d\chi_i(r)}{dr}|_{r=R_B}=-i k_i(R_B) - \frac{k_i'(R_B)}{2 k_i(R_B)} \quad (2.4)$$

Transformed in the “$\rho$-space” with $\rho = k \cdot r$, where $k$ is the asymptotic wave number

$$k=\sqrt{\frac{2\mu E}{\hbar^2}} \quad (2.5)$$

it has the form

$$f_i(\rho_B)=-\frac{d\chi_i(\rho)}{d\rho}|_{\rho=\rho_B}=-i K_i(\rho_B) - \frac{K_i'(\rho_B)}{2 K_i(\rho_B)} \quad (2.6)$$

with

$$K_i(\rho)=\frac{k_i(\rho)}{k}$$

$$=\sqrt{1-\frac{2\eta}{\rho} \frac{l(l+1)}{\rho^2} + \frac{V_0}{E} f(\rho) + i \frac{W_0}{E} g(\rho)} \quad (2.7)$$

Here $\eta$ is the Sommerfeld parameter:

$$\eta=\frac{\mu z_p z_T e^2}{\hbar^2 k} \quad (2.8)$$

The second kind of incoming waves that have been used in the literature [11] are the spherical waves

$$\chi_i(r)=e^{-ikkr} \quad (2.9)$$

where $K$ is a dimensionless parameter to be determined by a fit to the data. The logarithmic derivative which is needed to start the integration of the radial equation has in $\rho$-space the simple form

$$f_i(\rho_B)=-i K \quad (2.10)$$

Recently, Ohta and Okai [11] improved the $^{16}$O + $^{16}$O fits of Maher [4] with this model using a value of 4.4 fm for the boundary radius and $K=1.0$ for the logarithmic derivative. The incoming wave boundary condition can influence only those partial waves which have a nonvanishing amplitude at the boundary radius. These are the low partial waves up to

$$l$$