Quadrupole $P$-odd electron-nucleus interaction

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Received 18 April 1991; final version 28 June 1991

Abstract. The quadrupole distribution of the weak nuclear charge causes $P$-odd mixing of $s_{1/2}$ and $p_{3/2}$ atomic states. The measurement of the corresponding $P$-odd effects in atoms would allow to determine the neutron quadrupole moment of a nucleus. The experiments with rare earths are of a particular interest in this respect. A simple derivation is presented for the imitating effect which originates from the combined action of the total weak charge and the quadrupole hyperfine interaction.

PACS: 21.10.Gv; 31.30.Gs

1. Parity nonconservation in atoms is now a firmly established phenomenon. Its investigations bring a first-class quantitative information about the weak interactions of elementary particles (see, e.g., book [1]). Not only nuclear-spin-independent effects have been studied, but the first evidence has been obtained of much smaller nuclear-spin-dependent ones, caused by the so-called nuclear anapole moment [2]. Due to this progress it becomes reasonable to consider more subtle $P$-odd effects in atoms which depend on the nuclear spin, to be more exact, on the quadrupole distribution of the weak nuclear charge which is close to the quadrupole distribution of neutrons. These effects have been previously mentioned in [1].

One more proposal should be mentioned here [3]. The idea is, by using the accurate value of $\sin^2 \theta$ obtained at $Z$-boson peak at electron-positron colliders, to extract from the atomic experiments with different rare-earth isotopes the information on the neutron distribution in their nuclei.

2. The Hamiltonian of the $P$-odd electron-nucleon interaction which does not depend directly on nuclear spin is

$$H = -\frac{G}{V^{1/2}} \cdot \frac{Q}{2} \cdot \gamma_5 \cdot \rho (r),$$  

where $G = 10^{-5}/m_e^2$ is the Fermi weak interaction constant; $Q$ is the weak nuclear charge close numerically to $-N$, $N$ being the neutron number. The quadrupole component of the nuclear density $\rho (r)$ introduces in fact the dependence on nuclear spin into this interaction and causes the mixing of the electronic states $s_{1/2}$ and $p_{3/2}$. The relativistic wave functions behave at small $r$ as $r^{\gamma_2}$, where $\gamma_2 = (j+1/2)^2 - Z^2/4$. Therefore, the mixing discussed is suppressed as compared to the usually considered mixing of the states $s_{1/2}$ and $p_{1/2}$, due to the spherically-symmetric part of the nuclear density $\rho (r)$, by factor close to $Zr_0/a$ where $a/Z$ is the Bohr radius for the unscreened nucleus, $Z$ the nuclear charge, $r_0$ its radius. At $Z \sim 80$ this suppression constitutes about $10^{-2}$. Evidently, we lose here also the factor $Q$ as compared to the nuclear-spin-independent mixing. But some enhancement of the effect should occur in deformed nuclei.

3. We start our quantitative consideration from the case of deformed nuclei not only due to this enhancement, but because this case is slightly simpler technically. The density of nucleons in a nucleus can be considered with a good accuracy as a constant. Therefore, for spherical nuclei we can present this density as

$$\rho_0 (r) = \frac{3}{4 \pi r_0^3} \cdot \theta (r_0 - r).$$  

The constant density of a deformed nuclei with a spheroidal boundary is conveniently parameterized in its rest frame as [4]

$$\rho (r) = \rho_0 \left( \frac{r}{1 - \frac{\beta^2}{4 \pi} + \beta Y_{20}} \right).$$

Since the deformation $\beta$ is small, we can use the expansion of this expression:

$$\rho (r) = \rho_0 (r) + \frac{3}{4 \pi r_0^3} \cdot \beta Y_{20} \delta (r - r_0).$$
The nuclear rotation is fast as compared to the electron motion. So we have to average the quadrupole part of this expression over the rotation. This procedure is carried out at the fixed projection $\Omega$ of the nuclear spin $I$ onto the spheroid axis. As a result the quadrupole $P$-odd Hamiltonian is presented as

$$H_q = -\frac{G}{\sqrt{2}} \cdot \frac{Q}{2} \cdot \frac{3}{4 \pi r_0^2} \cdot \beta \delta (r-r_0) \cdot A \cdot \sum_m Y_{2m} S_{2m},$$

where $A = \frac{3 \Omega^2 - I(I+1)}{I(I+1)(2I-1)(2I+3)}$; $Y_{2m}$ are the spherical components of the tensor $\hat{I}_x \hat{I}_y + \hat{I}_y \hat{I}_z - \frac{3}{2} \delta_{ij} I(I+1)$; $S_{2m} = \frac{I}{r_0^2}(I+1)$.

It is convenient to choose the wave functions of the states $\rho_{3/2}$ and $\sigma_{1/2}$ as

$$\psi_{\rho_{3/2}} = \left( -f_{\rho}(r) \cdot (\sigma \alpha) \Omega_{3/2,2} \right);$$
$$\psi_{\sigma_{1/2}} = \left( g_{\sigma}(r) \cdot \Omega_{1/2,0} \right).$$

Here $\Omega_{jl}$ are the spherical wave functions with spin corresponding to the total angular momentum $j$ and orbital angular momentum $l$. Using for the radial wave functions $f$ and $g$ the following expressions at small $r$ (see, e.g., [1])

$$f_{njl} = -\frac{k}{|k|} \cdot (k-\gamma_{2j}) \left( \frac{Z}{a^3 \nu^3} \right) \frac{1}{2} \cdot \frac{2}{(2 \gamma_{2,j} + 1)} \left( \frac{a}{2 Zr} \right)^{1-\gamma_{2j}};$$
$$g_{njl} = \frac{k}{|k|} \cdot Zr \left( \frac{Z}{a^3 \nu^3} \right)^{1/2} \cdot \frac{2}{(2 \gamma_{2,j} + 1)} \left( \frac{a}{2 Zr} \right)^{1-\gamma_{2j}};$$
$$k = (-1)^{j+1/2-l}(j+1/2),$$

we get for the mixing matrix element

$$\langle \rho_{3/2} | H_q | \sigma_{1/2} \rangle = -ig \cdot \frac{Z^2 R_1}{(v_s v_p)^{3/2}} \cdot \frac{Q}{2} \cdot \nu \cdot \chi_{n,p} \cdot \frac{Zr_0}{a} \cdot \frac{R_{3/2}^{1/2}}{R_{1/2}^{1/2}} \cdot \frac{\beta}{6 \sqrt{5 \pi}} \cdot \frac{3 \Omega^2 - I(I+1)}{I(I+1)} \cdot \frac{\sqrt{I'(2I+3)}}{-\sqrt{(I+1)/(2I-1)}}.$$

Here $g = \frac{G m^2 a^2}{\sqrt{2 \pi}} = 3.65 \cdot 10^{-17}$; $m$ is the electron mass; $v_s$, $v_p$ are the principal quantum numbers of the $s$ and $p$ states; Rydberg $\text{Ryd} = ma^2/2$; $R_i$ and $R_3$ are relativistic enhancement factors:

$$R_{3/2}^{1/2} = \frac{4!}{\Gamma(2 \gamma_{3/2} + 1)} \cdot \left( \frac{a}{2 Zr_0} \right)^{2-\gamma_{3/2}},$$
$$R_{1/2}^{1/2} = \frac{2}{\Gamma(2 \gamma_{1/2} + 1)} \cdot \left( \frac{a}{2 Zr_0} \right)^{1-\gamma_{1/2}}.$$

The upper line in the curly bracket corresponds to the total atomic angular momentum $F = I + 1/2$, the lower to $F = I - 1/2$. In the second case the mixing is always larger. The best nuclear situation corresponds to $\Omega = I$, i.e., to a nonrotating nucleus.

In the first line of formula (8) we single out the factor equal to the mixing matrix element $\langle \rho_{1/2} | H_q | \sigma_{1/2} \rangle$. Then the second line is a kind of a suppression factor which constitutes about $10^{-4}$ at $\beta \sim 0.3$ and $Z \sim 60$.

If the nucleus is described by the shell model, the mixing matrix element for the atomic states is

$$\langle \rho_{3/2} | H_q | \sigma_{1/2} \rangle = -ig \cdot \frac{Z^2 R_1}{(v_s v_p)^{3/2}} \cdot \frac{Q}{2} \cdot \nu \cdot \chi_{n,p} \cdot \frac{Zr_0}{a} \cdot \frac{R_{3/2}^{1/2}}{R_{1/2}^{1/2}} \cdot \frac{\beta}{6 \sqrt{5 \pi}} \cdot \frac{3 \Omega^2 - I(I+1)}{I(I+1)} \cdot \frac{\sqrt{I'(2I+3)}}{-\sqrt{(I+1)/(2I-1)}}.$$

Again the upper line in the curly bracket corresponds to the total atomic angular momentum $F = I + 1/2$, the lower to $F = I - 1/2$. Here $\chi$ is the weak charge of the valence nucleon. It constitutes $\chi_n = -1/2$ for a neutron, and is much smaller numerically for a proton: $\chi_p = (1/2)(1 - 4 \sin^2 \theta) = 0.04$ at the experimental value of the mixing parameter $\sin^2 \theta = 0.23$. Even for a valence neutron the mixing constitutes about $10^{-5}$ only as compared to the nuclear-spin-independent effect.

Matrix element (10) was calculated under the assumption that the radial density of the valence nucleon follows the same law as the total one, i.e., is proportional to $\theta (r_0 - r)$. It simplifies the calculations since now they can be reduced by integration by parts again to the $\delta$-function kernel which in its turn allows us to use the usual Coulomb radial wave functions outside the nucleus. According to our estimates, the accuracy of this approximation is about 30–50%. If necessary, it can be easily improved in this respect by numerical calculations.

One should have in mind of course that in atoms with some outer electrons formulae (8) and (10) cannot be applied directly since at first we have to add the angular momenta of these electrons to form the total electronic one $J$ which in its turn couples to the nuclear spin $I$.

4. The effect discussed can be singled out in experiments on the parity nonconservation in atoms through its specific dependence on the total angular momenta $F, F'$ of the initial and final atomic states. Unfortunately, in the atoms being investigated now, caesium, thallium, bismuth, nuclei are not deformed and within the naive shell model the valence nucleon is a proton. So, the effect is