A Broken-Pair Description of $^{89}$Y, $^{91}$Nb and $^{93}$Tc

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The low-energy properties of $^{89}$Y, $^{91}$Nb and $^{93}$Tc are described in a broken-pair model. The shell model space for the protons consists of one major shell and for the neutrons particle-hole states within two major shells are taken into account. The effective interaction is assumed to be a simple Gaussian Serber force, which has proved to be the most successful in adjacent even nuclei.

Energy spectra up to about 3 MeV excitation energy and one-nucleon transfer data can be described very well. Also electromagnetic properties can be reproduced rather well if reasonable effective charges are used. No indication for deformed states, as found in Sn nuclei, is observed.

1. Introduction

For many years attempts have been made to describe the $N=50$ isotones. The main reason for this has been the supposition that $^{88}$Sr$_{50}$ might be treated as a proper inert core, so valence protons were restricted to the $2p\,^3/2$ and $1g\,^9/2$ single-particle levels. Within this small model space relationships between the energy spectra of several nuclei as predicted by the shell model can be shown to be satisfied to a large extent [1–6].

In several investigations it has been remarked however that also excitations from the $2p\,^3/2$ level are important, especially to describe electromagnetic properties [7–12]. In the work of Vergados and Kuo [11] both proton and neutron excitations from the $^{88}$Sr core were allowed for the description of the energy levels of $^{89}$Y. In a recent, rather extensive study of both the even and odd $N=50$ isotones Fujita and Komoda allowed one - or two-proton excitations from the $2p\,^3/2$ level [12]. Although this is sufficient to obtain non-zero values for $M1$, $E2$ and $E3$ transitions, the comparison with experimental data points out that the results are not always satisfactory, especially for collective transitions in even nuclei. Such collective transitions have been studied extensively by Gillet et al. [13] in the framework of the BCS model. They found that the $E3$ excitations can be reasonably well accounted for by this model, but the description of $E2$ excitations was found to be poor.

Allaart and Boeker have demonstrated however that the latter are considerably improved for $N=50$ isotones by a systematic particle-number conserving BCS treatment [14].

There are several reasons why one may prefer a number-conserving BCS quasiparticle model [15], which is equivalent with a broken-pair model [16] or the generalized seniority scheme [17], rather than a straightforward shell model treatment. One reason is that one can easily deal with more than one major shell to describe the pairing properties [18]. Also when one restricts the model to one major shell the projected quasiparticle or broken-pair model yields a good prescription how to select a few model states out of a many times larger shell model basis without loosing much accuracy [19]. It also provides a transparent picture of the structure of the nuclear states, supposing that most nucleons occur as unbroken pairs (which are supposed to be the microscopic equivalent of $S$-bosons in [20]).

In the present paper we report the application of the projected quasiparticle [15] or broken-pair [16] model to odd $N=50$ isotones. So far this model has

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been applied to odd Sn nuclei only [21, 22]. Since it is known [23] that in Sn nuclei certain deformed structures may appear at rather low excitation energy, the \( N = 50 \) isotones might be more suitable for the application of the model. As the description of the odd nuclei can only be expected to be successful when also the lowest (collective) excited states can be described by the same method, these states are considered first, and improved by the inclusion of neutron particle-hole excitations. Therefore the basis states for the description of the odd isotones are states with one broken proton pair or a neutron particle-hole pair. In Sect. 2 the model and the computational method are outlined. Section 3 contains the resulting spectroscopic properties and a comparison with experimental data. Section 4 contains a summary and conclusions.

2. Model and Computational Procedure

2.1. The Model Space

The model space consists of states of the following three types

a) states without a broken pair:

\[ a^+_\alpha (S^+)^\alpha |\bar{\alpha}\rangle \]  

(2.1)

b) states with a broken proton pair:

\[ a^+_\alpha a^-_\beta (S^+)^{\alpha - 1} |\bar{\alpha}\rangle \]  

(2.2)

c) states with a neutron particle-hole pair:

\[ a^+_\alpha b^+_\mu b^-_\nu (S^+)^\mu |\bar{\alpha}\rangle \]  

(2.3)

where \( |\bar{\alpha}\rangle \) denotes the closed shell \((Z = 28, N = 50)\) state and \( S^+ = \sum_\alpha \frac{\hat{C}_\alpha}{2} \mu_\alpha^a A_{500}(aa) \) in obvious notation [19]. The BCS-parameters \( v_\alpha, u_\alpha \) are determined such that the presence of unpaired particles is accounted for in an average way [23]. The technique how to calculate matrix elements of a shell model hamiltonian in the space of states (2.1) and (2.2), coupled to proper angular momenta, is well known [15, 21]. In [15] extensive formulas have been given. The extension to include the particle-hole states (2.3) is straightforward. Formulas are given in the appendix.

The shell model orbits included are the \( \text{f}_5/2, \text{p}_3/2, \text{p}_1/2 \) and \( \text{g}_9/2 \) shells for protons, the \( \text{f}_5/2, \text{p}_3/2, \text{p}_1/2 \) and \( \text{g}_9/2 \) for the neutron hole and the \( \text{d}_5/2, \text{d}_3/2, \text{s}_1/2 \) and \( \text{h}_{11/2} \) shells for the neutron particle. A restriction, which was justified by some test calculations, is that the neutron particle-hole configuration should have natural parity \((J^p = 2^+, 3^-, 4^+, \ldots)\). The \( J^p = 1^- \) configurations were excluded because they only produce a low-lying spurious (center of mass motion) state and high-lying states which do not contribute significantly to the lowest part of the spectra.

2.2. The Model Parameters

The shell model effective interaction is assumed to be of the form

\[ V(1,2) = - V_0 R_2 \exp \left( \frac{-r^2}{\mu^2} \right) \]  

(2.4)

where \( R_2 \) is the singlet operator. The range parameter \( \mu \) is taken to be 1.9 fm. Although this force is very simple it appears to give good results in practical calculations of even single-closed-shell nuclei, better than several more complicated forces [24]. The proton single-particle energies were determined from experimental data by the number-conserving analogue of the inverse gap equations [25]. In \( ^{91}\text{Nb} \) these energies were taken slightly different from those of [25] in order to reproduce the fragmentation of the \( \text{p}_3/2 \) and \( \text{f}_5/2 \) single-particle strength better. For the neutron single-particle energies a reasonable guess was made. The results are not sensitive to small changes in these energies. The parameters are listed in Table 1. For the even nuclei, of which we need the ground state wave function for the calculation of one-nucleon transfer spectroscopic factors, interpolated parameters are used. Further model parameters are the effective charges \( e_{\text{eff}} = 1.4e \) for protons and \( 0.4e \) for neutrons and the effective spin-gyromagnetic factors \( g_{\text{eff}} = 3.29 \text{n.m.} \) for protons and \(-2.50 \text{n.m.} \) for neutrons. With these values we reproduced the \( B(E2;2^-\rightarrow g.s.) \) and \( B(M1;1^-\rightarrow g.s.) \) in \( ^{88}\text{Sr} \).

3. Results and Discussion

3.1. Spectroscopic Factors

Table 2 shows that the experimental values for one proton transfer reactions are reproduced reasonably well by the calculation. (The pick-up data for \( ^{89}\text{Y} \)}...