Muon polarization in the decay $K_L \rightarrow \bar{\mu} \mu$
in superstring inspired models

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Abstract. In superstring models, contributions to the muon longitudinal polarization $P_L$ in the decay $K_L \rightarrow \bar{\mu} \mu$ can arise from interactions involving the exotic $D$ and $D^*$ superfields. It is shown that these interactions also contribute to the decay $\mu \rightarrow e \gamma$. After using the experimental limit to constrain this process, we find that $P_L$ arising from these new interactions is less than $10^{-4}$.

The first evidence that CP is not an exact symmetry in the $K^0$ decay was discovered in 1964 [1]. The observed CP violation parameter, $\epsilon$, has a value of $2 \times 10^{-3}$. Shortly after the discovery of CP violation, it was pointed out by Lincoln Wolfenstein that CP violation in the $K^0$ system could be explained by a superweak interaction [2]. This simple phenomenological model predicts that null-results will be found in other experiments, and 23 years later, the observed CP violation is still amazingly confined to the $K_L - K_S$ system. Ideas of probing other CP violation is still amazingly confined to the $K_L - K_S$ system. Ideas of probing other CP violation observables have been suggested [3]. In particular, it was realized by Herczeg [4] that CP violation beyond the superweak idea would exist if the muon longitudinal polarization, denoted by $P_L$ hereafter, in the decay $K_L \rightarrow \bar{\mu} \mu$ is greater than the order of $10^{-3}$, i.e. greater than the effect arising from the $K_L - K_S$ mixing. To calculate $P_L$ we follow Herczeg and write the effective Hamiltonian as

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} (\bar{\bar{\tilde{\gamma}}}_S d)(a \bar{\mu} \gamma_5 \mu + i b \bar{\mu} \mu) + \text{h.c.}$$

(1)

In general $a$ is complex because it includes the standard absorptive part from the $\gamma \gamma$ intermediate state whereas the CP-violating piece $b$ is real. The muon longitudinal polarization is given by

$$P_L = \frac{N_L - N_R}{N_L + N_R},$$

(2)

where $N_{L,R}$ represents the number of the left- and right-handed final state muons. Using the known value of $Im \epsilon$ and the experimental rate one finds

$$P_L = 1.8 \times 10^{-3}.$$  

(3)

Calculations for $b$ and thus $P_L$ in the standard electroweak model [5] as well as in its supersymmetric extension models [6], in the left-right model [7] and in the spontaneous CP violation Higgs Model [8] have been carried out. While in the spontaneous CP violation Higgs model we find that the order-of-magnitude of $P_L$ lies in the region between the order of unity and $10^{-2}$, in the other models, $P_L$ always turns out to be less than $10^{-3}$.

In this paper we wish to examine $P_L$ in the context of superstring [9] inspired models. The most attractive superstring scenario is based on the $E_8 \times E_8$ heterotic string [10] in 10-dimensions that upon compactification leads to a 4-dimensional $E_6$ grand unified $N=1$ supergravity theory [11]. Matter fields in this $E_6$ gauge group belong to a $\{27\}$ representation that under the $[SO(10), SU(5)]$ subgroups has a decomposition

$$\{27\} = \{16,10\} + \{16,5\} + \{16,1\} + \{10,\bar{5}\} + \{10,5\} + \{1,1\},$$

(4)

where

$$Q = \begin{pmatrix} u \\ d \end{pmatrix}, \quad L = \begin{pmatrix} \nu \\ \ell \end{pmatrix}, \quad H = \begin{pmatrix} H^+ \\ H_0 \end{pmatrix}, \quad H = \begin{pmatrix} H^0 \\ H^- \end{pmatrix}. $$
In this model the most general low-energy superpotential contains 11 terms. Suppressing the generation indices we have

$$\alpha_1 Q d^c \bar{H} + \alpha_2 Q u^c H + \alpha_3 L e^c \bar{H} + \alpha_4 L v^c H + \alpha_5 \bar{H} H N + \alpha_6 D D^c N,$$

and

$$\beta_1 DQQ + \beta_2 D^c u^c d^c.$$

and

$$\lambda_1 D^c L Q + \lambda_2 D e^c u^c + \lambda_3 D v^c d^c.$$

The presence of both (6) and (7) will lead to an undesired fast proton decay [12]. It turns out, however, that (6) and (7) conserve the baryon number separately. An interesting scenario is to exclude (6) from the superstring formalism by introducing a $Z_2 \times Z_3$ discrete symmetry [13], although no superstring model constructed has this particular symmetry. Under this discrete symmetry the term $\alpha_5 L v^c H$ that provides the neutrino with a tree level mass is also forbidden. It has been shown recently [14-16] that this model can provide a simple and natural way of generating small Dirac neutrino masses through radiative corrections. Hereafter we will concentrate our efforts on the discussion of the muon longitudinal polarization in this model.

An interesting feature of this model is the presence of the exotic heavy particles $D$ and $D^c$ which interact with the quark and the lepton through (7) like leptoquarks. Also, in contrast to the minimal supersymmetric standard electroweak model [17], in which the additional CP violation phase beyond the one appearing in the KM model [18] is severely constrained by the gluino contribution to the neutral electric dipole moment [19], other CP-violating phases can arise in this model, for instance, from complex coupling constants $(\lambda_i)_{mn}$, here $m$ and $n$ are the generation indices, and soft-supersymmetry breaking terms. Given a large number of independent parameters, it is conceivable [20] that not all of these phases are constrained by the known phenomenology. In any case, only if we would get a large value of $P_L$ that is of experimental interest ($P_L \gg 10^3$) for a reasonable choice of parameters, then we would have to check if the associated CP-violating phase were constrained by other physical processes.

At one-loop level, apart from those diagrams given by the minimal supersymmetric standard model discussed in Ref. 6, additional graphs which contribute to $P_L$ can arise from the intermediate $D$ and $D^c$ exchanges. The dominant contribution to $P_L$ is shown in Fig. 1. Evaluating the graph in Fig. 1 (there are 4 similar graphs and only one of them is displayed in Fig. 1), we find

$$b_w = \frac{\xi}{12 \pi^2} |(\lambda_1)_{22}(\lambda_2)_{22}| \frac{m_e}{M} \left(\frac{M_w}{M}\right)^2 \sin \theta_c \sin \delta,$$

where we have omitted the index that labels different generation exotic $D$ and $D^c$ fields, $\theta_c$ is the Cabibbo angle arising from the $\bar{W} - d_L - \bar{e}_L$ vertex, and $\delta$ is a combination of CP-violating phases appearing in the diagram. For simplicity, in evaluating Fig. 1 we have assumed that all the superparticle masses are approximately the same. We denote this mass by $M$. The order-of-magnitude of $M$ should be given by the supersymmetry breaking scale which could be, for instance, of the order $M_w$. In (8), $\xi$ is the $\bar{e}_L - \bar{e}_R$ mixing arising from the soft-supersymmetry breaking terms, and for a reasonable choice of the parameters its value is roughly (see [17])

$$\xi \sim \frac{m_e}{M}.$$  

Combining (8) and (9) yields

$$b_w \sim \frac{1}{12 \pi^2} |(\lambda_1)_{22}(\lambda_2)_{22}| \frac{m_e m_e}{M^2} \left(\frac{M_w}{M}\right)^2 \sin \theta_c \sin \delta.$$  

To estimate the upper limit for the $\lambda$ couplings in (10), we find that the most relevant process is the decay $\mu \rightarrow e\gamma$ generated by the graphs shown in Fig. 2.