Mixing of Intermediate Bosons with $Q\bar{Q}$ Bound States

F.M. Renard
Département de Physique Mathématique*, Université des Sciences et Techniques du Languedoc, F-34060 Montpellier Cedex, France

Abstract. We discuss the effects of a close degeneracy between the $Z^0$ and vector $Q\bar{Q}$ bound states; large mixing effects can appear modifying mainly the $Q\bar{Q}$ states (widths and couplings) and leading to curious structures inside the $Z$ peak.

Search for heavy $Q\bar{Q}$ vector mesons will be one item of the program of the future LEP machine [1,2]. At least one such series is strongly expected, that one formed with top quarks ($t\bar{t}$) but other types of quarks could well exist too. Standard quarkonia models let us expect a repetition of the $\psi$ and $Y$ cases with a series of narrow states and a threshold enhancement filled with broader resonant states. We now consider the case where such a series lies near the $Z^0$ intermediate boson. The coupling of the $Z^0$ to the $Q\bar{Q}$ vector state $V^0$ should be similar to the one of the photon (already well-known for $\rho, \omega, \phi, \psi, Y, \ldots$) but the vicinity of the masses $m_{+}$ and $m_{0}$ is a source of strong mixing effects very specific of such a situation and this is what we discuss below.

Before let us just notice first that in principle a similar mixing could also well appear between $Z^0$ and $Q\bar{Q}$ axial vector states $A^0$ but as the $1^{++}$ bound state wave function at the origin is much weaker than the vector one mixing effects are expected to be much smaller too; secondly the following $Z^0 - V^0$ analysis can be exactly extended to the charged case $W^\pm - V^\mp (Q\bar{Q})$ provided that $m_{\pm} = m_{\mp}$.

The main source of $Z^0 - V^0$ mixing is due to the process of Fig. 1 (high order strong or electromagnetic terms are neglected). The $Z^0 - V^0$ vertex is similar in nature to the $\gamma - V^0$ vertex controlled by the value at the origin of the bound state wave function:

$$e g_{V^0\gamma} = 2e_q \sqrt{m_{\nu} \phi_{V^0}(0)}$$

$$e g_{V^0\gamma} = 2e \left(1 - \frac{3}{2} \sin^2 \theta_w \right) \sqrt{m_{\nu} \phi_{V^0}(0)}$$

For $m_{\nu} \approx 100 \text{ GeV}$, this leads to $g_{V^0\gamma} \approx 150 \text{ GeV}^2$ and $g_{V^0\gamma} \approx 100 \text{ GeV}^2$ for fundamental $Q\bar{Q}$ states. It is also possible that at high masses the effective $Q\bar{Q}$ potential becomes relatively more coulombic at short distances and consequently that the (hydrogenic) wave function increases more like $-\frac{3}{2} m_{\nu}$ near the origin such that $g_{V^0\gamma}$ could be slightly larger.

In order to quickly estimate mixing effects we use now the simple mass-mixing model already very successful in other vector meson problems [4]. Starting from unmixed $Z^0, V^0$ states we diagonalize the complex mass-matrix:

$$\begin{pmatrix}
    m_{Z^0} - \frac{i}{2} \Gamma_{Z^0} & \delta m_{Z^0\nu} \\
    \delta m_{Z^0\nu} & m_{\nu} - \frac{i}{2} \Gamma_{\nu}
\end{pmatrix}$$

using the complex mixing angle $\theta$ given by $\tan 2\theta = \frac{2\delta m_{Z^0\nu}}{m_{Z^0} - m_{\nu} - \frac{i}{2} (\Gamma_{Z^0} - \Gamma_{\nu})}$.
The physical states are now:
\[ |Z\rangle = \cos \theta |Z^0\rangle + \sin \theta |V^0\rangle \]
\[ |V\rangle = -\sin \theta |Z^0\rangle + \cos \theta |V^0\rangle \]  
(3)

their coupling constants to decay channels \( f \) become
\[ g_{zf} = \cos \theta g_{z0f} + \sin \theta g_{v0f} \]
and
\[ g_{vf} = -\sin \theta g_{z0f} + \cos \theta g_{v0f} , \]
(4)

and the physical masses and widths become:
\[ m_Z - \frac{i}{2} \Gamma_Z = \left( m_{Z^0} - \frac{i}{2} \Gamma_{Z^0} \right) \cos^2 \theta \]
\[ + \left( m_{V^0} - \frac{i}{2} \Gamma_{V^0} \right) \sin^2 \theta + \delta m_{zv0} \sin 2 \theta \]
\[ m_V - \frac{i}{2} \Gamma_V = \left( m_{V^0} - \frac{i}{2} \Gamma_{V^0} \right) \cos^2 \theta \]
\[ + \left( m_{Z^0} - \frac{i}{2} \Gamma_{Z^0} \right) \sin^2 \theta - \delta m_{zv0} \sin 2 \theta \]  
(5)

Any amplitude (for example \( e^+ e^- \to (Z + V) \to f \)) passing through \( Z \) and \( V \) mesons will be written [5]:
\[ R_f = \frac{g_{z0} g_{zf} + g_{v0} g_{vf}}{D_Z} \]
(6)
with the propagators \( D = s - m^2 + i m \Gamma \).

The mixing term is given by the \( Z^0 - V^0 \) transition:
\[ \delta m_{zv0} = \frac{e \delta g_{zv0}}{2 m_{v0}}, \]
(7)

it can be complex if physical intermediate states \( n \) can be inserted between the \( Z^0 \) and the \( V^0 \) (for example when \( V^0 \) lies above the threshold for \( Q\bar{Q} \) like mesons), in such a case:
\[ \text{Im} \delta m_{zv0} \simeq -\frac{i}{2} \Gamma_{Z^0\rightarrow n}^{1/2} \Gamma_{V^0\rightarrow n}^{1/2} \]  
(8)

From the process of Fig. 1 one expects \( \text{Re} \delta m_{zv0} \simeq 0.15 \) GeV for low-lying \( Q\bar{Q} \) vector states. For excited states the wave function at the origin and consequently \( \text{Re} \delta m_{zv0} \) become smaller; above the \( Q\bar{Q} \) threshold an imaginary part is developed proportionally to the total width \( \Gamma_{Z^0}^{1/2} \) and a reasonable order of magnitude seems to be \( \text{Im} \delta m_{zv0} \simeq 0.1 \text{ GeV} \). So from Eq. (3), using \( \Gamma_{Z^0} \simeq 2.7 \text{ GeV} \) and \( \Gamma_{v0} \ll \Gamma_{Z^0} \) it appears that an appreciable mixing effect can appear only if \( |m_{Z^0} - m_{V^0}| \ll \Gamma_{Z^0} - \Gamma_{V^0} \). In the best case where \( m_{Z^0} \approx m_{V^0} \) we get \( |\theta| \approx 0.2 \) which is not negligible at all. If \( m_{Z^0} \) is far from \( m_{V^0} \), \( \theta \to 0 \) and the \( Z \) and \( V \) amplitudes can be separately considered as unmixed \( Z^0 \) and \( V^0 \) ones.

In the following we consider explicitly the case where \( m_{Z^0} \approx m_{V^0} \) and discuss the effect of the vicinity of the \( Z^0 \) on the whole \( Q\bar{Q} \) vector spectrum.

In the normal case \( |\theta| \ll 0.2 \), from Eq. (5) we get physical masses \( m_Z \) and \( m_V \) differing at most by \( 0.1 \) GeV from the bare masses \( m_{Z^0} \) and \( m_{V^0} \). If the \( V^0 \) width is very small (i.e. tens of keV as expected from QCD for low-lying states) then the physical \( Z \) width will also be only slightly affected by the mixing, whereas on the opposite the \( V \) width is enlarged roughly up to \( \Gamma_V \approx |\theta|^2 \Gamma_{Z^0} \lesssim 0.1 \) GeV. Let us also see what happens in specific channels using Eq. (4). The couplings of \( Z \) to leptons and light quarks will effectively be weakly modified because both
\[ \left( g_{v0} \gamma_\mu - \frac{e^2}{m_{v0}^2} \right) \approx \left( g_{z0} \gamma_\mu - \frac{e^2}{m_{z0}^2} \right) \approx \left( g_{z0} \gamma_\mu - \frac{e^2}{m_{z0}^2} \right) \approx \left( g_{z0} \gamma_\mu - \frac{e^2}{m_{z0}^2} \right) \]
and
\[ (g_{v0} \gamma_\mu - \gamma_\mu \gamma_5) \approx (g_{z0} \gamma_\mu - \gamma_\mu \gamma_5) \]

On the opposite the \( V \) couplings are strongly enhanced, roughly \( g_{v0} \gamma_\mu - \gamma_\mu \gamma_5 \). In addition a large parity violation is induced because they get an axial coupling \( g_{v0} \gamma_\mu \gamma_5 \) which can be (similarly to \( \psi^{(0)} \) states) of several tens of MeV; only the couplings to heavy quarks \( Q\bar{Q} \) will have a different behaviour as now the bare couplings \( g_{v0} \gamma_\mu \) and \( g_{z0} \gamma_\mu \) may have comparable magnitudes and appreciable modifications in phase and magnitude appear through Eq. (4). In the (very improbable) limiting case where both \( m_{Z^0} \approx m_{V^0} \) and \( \Gamma_{V^0} \approx \Gamma_{Z^0} \), one would get independently of \( \delta m_{zv0} \) the limiting mixing angle