Real-space renormalization group investigation of pure and disordered mixed spin Ising models on \( d \)-dimensional lattices*

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Received March 6, 1990

The mixed spin Ising model (spins \( \sigma = \pm \frac{1}{2} \) and \( S = 1 \)) on \( d \)-dimensional hypercubic lattices with nearest-neighbour exchange interactions is studied via a renormalization group transformation in position space. The phase diagrams in \((L, K)\) space, i.e. in dependence of the bilinear \((K)\) and the biquadratic \((L)\) interaction coefficients, are qualitatively different for \( d = 2 \) and \( d > 2 \). For any dimension \( d \) however it is found that all transitions are of second order. At zero-temperature \((K = \infty, L = \infty)\), the ferromagnetic order disappears at \( \langle L/K \rangle_0 = 2 \), which does not depend on \( d \). Using an extension of this real-space renormalization group analysis we study the two-dimensional random disordered version of the above model. \( L \) is kept homogeneous and the bilinear interactions \( K_{ij} \) are assumed to be independent random variables with distribution \( P(K_{ij}) = p \delta(K_{ij} - K) + (1 - p) \delta(K_{ij} + K) \); where \( K > 0 \). The phase diagrams for different values of \( p \) are obtained. At zero temperature, it is found that in the bond diluted model \((x = 0)\) the value \( \langle L/K \rangle_0 \) depends continuously on \( p \), whereas in the random \( \pm K \) interactions \((x = -1)\) \( \langle L/K \rangle_0 \) is unique and does not depend on \( p \).

I. Introduction

The biquadratic exchange interaction \([1-5]\) and the single ion anisotropy \([4-7]\) has been pointed out to give significant effects on the Curie temperature and other magnetic properties of ferromagnets. In all these investigations, the spins had the same value on each lattice site. Therefore it is interesting now to extend the investigation to pure and random mixed spin-\( \frac{1}{2} \) and spin-1 Ising models defined on \( d \)-dimensional hypercubic lattices and described by the following reduced Hamiltonian:

\[
\beta H = - \sum_{\langle ij \rangle} K_{ij} \sigma_i \sigma_j + \sum_{\langle ij \rangle} L_{ij}(\sigma_i \sigma_j)^2.
\]

The underlying lattice is composed of two interpenetrating sublattices, one occupied by spins with spin moment \( \sigma = \pm \frac{1}{2} \) and the other one is occupied by spins with spin moment \( S = 0, \pm 1 \). The summation in \((1)\) extends over all pairs of nearest-neighbour sites of the lattice. \( K_{ij} \) and \( L_{ij} \) denote the reduced bilinear and biquadratic exchange interactions, respectively. For isotropic \( L_{ij}(L_{ij} = L) \) one should note that the Hamiltonian \((1)\) is equivalent to a mixed spin Ising system with crystal field interaction

\[
\beta H = - \sum_{\langle ij \rangle} K_{ij} \sigma_i \sigma_j + \frac{d}{2} \sum_j L_j S_j^2
\]

where \( d \) is the dimensionality of the hypercubic lattice.

In the simplest form of \((1)\), namely \( K_{ij} = K \) and \( L_{ij} = 0 \), the above model has been studied by renormalization group techniques \([8]\) (in two dimensions) and high-temperature series expansions \([9]\). It has been shown that the critical exponents are in good agreement with those suggested by universality ideas. Recently, attention has been directed to the pure model \((K_{ij} = K, L_{ij} = L)\) described by the Hamiltonian \((1)\). The effects of the biquadratic exchange interaction on the Curie temperature has been discussed \([10-12]\) by the use of the Bethe-Peierls method. It was shown that this extra-term significantly changes the value of the transition temperature but it was not clarified whether the system exhibits a tricritical behaviour or not. Using an effective-field approximation \([13]\) and the finite-cluster approximation \([14]\), it was shown that a tricritical point exists if the coordination number \( z \) is larger than 3.

The first purpose of this paper is to study the pure \( d \)-dimensional mixed spin Ising model described by the Hamiltonian \((1)\). This investigation is performed using the Migdal-Kadanoff renormalization group method

* Supported by the agreement of cooperation between the DFG-W. Germany and the CNR-Maroc

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In addition, we study the existence of tricritical behaviour for any dimension $d$ of the system. We remark that the method we employ can not be restricted to a mixed spin Ising model with only bilinear term ($L=0$), since in the two-dimensional parameter space ($L, K$), the subspace $L=0$ is not invariant.

Several authors used real-space renormalization group techniques to study the magnetic properties of Ising models on monoatomic lattices with quenched bond dilution [17–20]. The same model with a biquadratic term (or with crystal field) has also been investigated by various methods [6, 7]. It was shown that this extra term modifies significantly the phase diagram.

The second purpose of this work is to present an extension of this study to randomly disordered mixed spin-½ and spin-1 Ising models. We investigate this model using an extension of the Migdal-Kadanoff transformation to disordered systems. The reduced bilinear interactions $K_{ij}$ are assumed to be independent random variables with distribution

$$P(K_{ij}) = p \delta(K_{ij} - K) + (1 - p) \delta(K_{ij} - aK),$$

where $K > 0$. The biquadratic exchange interactions $L_{ij}$ are kept constant and homogeneous

$$Q(L_{ij}) = \delta(L_{ij} - L).$$

In Sect. 2 phase diagrams of the $d$-dimensional pure model are determined within the framework of a real-space renormalization group (RSRG) technique. Using an extension of this method in Sect. 3, we determine the phase diagrams for both kinds of disorder. In particular we investigate the influence of dilution and the influence of random ±$K$ interactions on the critical value $\left(\frac{L}{K}\right)_0$ at zero-temperature.

II. Pure system

The different phases of the model described by Hamiltonian (1) can be characterized by two parameters: the magnetization $m$ (or $m_\sigma$) and the quadrupole parameter $\bar{q}$:

$$m(K, L) \equiv \langle S_\sigma \rangle = Z^{-1} \sum_{\sigma, S} S_\sigma \exp(-\beta H),$$

$$\bar{q}(K, L) \equiv \langle S^2_\sigma \rangle = Z^{-1} \sum_{\sigma, S} S^2_\sigma \exp(-\beta H),$$

where the sums are over all spin configurations and $Z$ is the partition function. Since the $\sigma$- and $S$-sublattices are translationally invariant, $m$ and $\bar{q}$ do not depend on the site $i$. According to the values of $m$ and $\bar{q}$, three different phases can be distinguished as follows:

- Paramagnetic ($P_-$): $m=0$, $\bar{q}<\frac{1}{2}$
- Paramagnetic ($P_+$): $m=0$, $\bar{q}>\frac{1}{2}$
- Ferromagnetic ($F_-$): $m=0$, $\bar{q}\geq\frac{1}{2}$

In terms of the renormalization group theory, these phases and all transitions are determined by the topology of the RSRG flow linking the various fixed points. In this section we determine the phase diagram in $(L, K)$ space for $d$-dimensional hypercubic lattices using the Migdal-Kadanoff renormalization group method. This renormalization technique is applicable for all space dimensionalities. We give the recursion relations for any $d$ after briefly describing the method.

Because of the symmetry of the model, we have to restrict ourselves to an odd scale factor $b$. In the present study we choose $b=3$ and consider a one-dimensional chain with spins $\sigma_1, \sigma_2, \sigma_3$. All these spins are coupled by the same bilinear and biquadratic exchange interaction $K$ and $L$, respectively. The corresponding reduced Hamiltonian reads

$$\beta H = -K(\sigma_1 S_1 + \sigma_2 S_2 + \sigma_3 S_3) + L(\sigma_1^2 S_1^2 + \sigma_2^2 S_2^2 + \sigma_3^2 S_3^2).$$

Performing the trace over $S_3$ and $\sigma_3$ we obtain the transformed reduced Hamiltonian

$$\beta \bar{H} = -\bar{K} \sigma_1 S_1 + \bar{L} \sigma_1^2 S_1^2.$$

The renormalized one-dimensional $\bar{K}$ and $\bar{L}$ are given as functions of $K$ and $L$ by

$$\bar{K} = 3^{d-1} K(R, L, K),$$

$$\bar{L} = 3^{d-1} L(R, L, K).$$

The Migdal-Kadanoff recursion relations for the $d$-dimensional hypercubic lattice are obtained using the usual moving procedure, as illustrated in Fig. 1. They are given by

$$K' = 3^{d-1} \bar{K}(R, L, K),$$

$$L' = 3^{d-1} \bar{L}(R, L, K).$$

These relations have been obtained by first performing the trace and then moving the bonds. We could have first moved the bonds and then performed the trace.