Two circular inclusions with inhomogeneous interfaces interacting with a circular Eshelby inclusion in anti-plane shear

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Summary. An analytical solution in infinite series form for two circular cylindrical elastic inclusions embedded in an infinite matrix with two circumferentially inhomogeneous imperfect interfaces interacting with a circular Eshelby inclusion in anti-plane shear is derived by employing complex variable techniques. All of those coefficients in the series can be uniquely determined in a simple and transparent way. Numerical examples are given to illustrate the effect of imperfection and circumferential inhomogeneity of the two interfaces as well as the size, location and elastic properties of the two circular inclusions on the stress fields induced within the two circular inclusions and the Eshelby inclusion.

1 Introduction

In recent years, elastic inclusions with imperfect interface are becoming an important research topic in the micro-mechanical analysis of composite materials. Here, the imperfect interface model is based on the assumption that tractions are continuous across the interface while displacement jumps are proportional, in terms of the spring-factor type parameters, to their traction components. When the spring-factor type parameters are constant along the entire interface, the interface is called a homogeneously imperfect interface; while when the spring-factor type parameters are varied along the interface, the interface is termed a circumferentially inhomogeneous imperfect interface. Ru and Schiavone [1] analyzed the problem of an isolated circular inclusion with circumferentially inhomogeneous imperfect interface under anti-plane shear deformations. Their results show that the circumferentially inhomogeneous imperfection of the interface will exert a prominent influence not only on the nonuniform stress field but also on the average stress induced within the circular inclusion. For the special homogeneously imperfect interface, they found that the stress field within the circular inclusion under remote uniform anti-plane shearing is still uniform. Shen et al. [2] considered the problem of an elliptical inclusion with homogeneously imperfect interface in anti-plane shear. Their results show that the interface imperfection has a significant effect on the stress fields in and near the inclusion. The nonuniformity of stresses is closely related to the interface parameter describing the imperfection and the aspect ratio of the ellipse. Ru [3] presented a rigorous solution for a circumferentially inhomogeneous sliding interface in plane elastostatics. He discussed the effects of the circumferential variation of the interface parameter on the mean stress at the interface and the average stresses within the inclusion. Sudak et al. [4] solved the problem of a circular inclusion with inhomogeneously imperfect interface in plane elasticity.
In their discussion, the imperfect interface is characterized by one in which there is a displacement jump across the interface in the same direction as the corresponding tractions, and the same degree of imperfection is realized in both the normal and tangential directions. Their results show that replacing the inhomogeneous interface by its homogeneous counterpart will lead to significant errors in even the calculation of the average stresses induced within the inclusion. Gao [5] also investigated a circular inclusion with imperfect interface in plane elasticity. He found that the stress field within the inclusion is no longer uniform. Zhong and Meguid [6] examined a spherical inclusion with imperfect interface bonded to an infinite matrix, and they also found that the stresses inside the inclusion of an imperfectly bonded interface are not uniform.

All of the aforementioned works have focused primarily on a solitary inclusion with imperfect interface and neglected the perturbation caused by the neighboring inclusions. The work of Kouris [7] reveals that both the degree of interfacial integrity, which is characterized by a spring-type thin layer, and the relative distance between fibers (volume fraction) have a considerable effect on stress concentration. As a result, the “single fiber” solution is quite inaccurate in predicting the local stress field, which is essential from a fracture point of view. Based on these considerations, the present work investigates two circular inclusions with circumferentially inhomogeneous imperfect interfaces interacting with a circular Eshelby inclusion in anti-plane shear. The term Eshelby inclusion is specially introduced here to indicate a subdomain which undergoes uniform eigenstrains and which possesses the same elastic properties as those of the surrounding matrix. A recent survey and bibliography on this subject can be found in the papers of Ru [8], [9] who derived analytic solutions for an arbitrarily shaped Eshelby inclusion in a plane or half-plane in terms of some auxiliary functions. As far as we know, no attempt has been made to analyze the aforementioned interesting problem, despite its intimate relevance to composite mechanics. Through employing complex variable techniques, we obtain an analytical solution in series form to this problem. All of those coefficients in the series can be uniquely determined in a simple and transparent way. It shall be stated that in treating the circular Eshelby inclusion, the same method suggested by Ru [8], [9] is adopted in our analysis. Our specific calculations illustrate that imperfection and circumferential inhomogeneity of the interfaces as well as geometry parameters and elastic constants of the two circular inclusions will exert a prominent influence on the stress fields induced within the two circular inclusions and the circular Eshelby inclusion.

2 Problem statement and basic formulation

Let us consider an infinite elastic matrix containing two circular inclusions with different elastic properties, as shown in Fig. 1. The left elastic inclusion with shear modulus \( \mu_3 \) occupies the circular region \( S_3 \) whose radius is unit 1. The right elastic inclusion with shear modulus \( \mu_1 \) occupies another circular region \( S_1 \) whose radius is \( (x_1 - x_2)/2 \). The distance between the centers of the two circular inclusions is \( (x - 1 + x_2)/2 \). The matrix with shear modulus \( \mu_2 \) occupies the region \( S_2 \). In addition, we further assume that a circular subdomain \( S_0 \) within the matrix, whose center and radius are respectively \( z_0 \) and \( R_0 \) as shown in Fig. 1, undergoes uniform eigenstrains. Since the subdomain and the matrix possess the same elastic properties, it will be specially termed an Eshelby inclusion in the present analysis. In the present paper, we confine our attention to anti-plane shear deformation, then it suffices to discuss the out-of-plane displacement component \( w(x, y) \) and the associated stress components in anti-plane shear...