A Study of Correlation Expansions of Field Theoretic Amplitudes

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Received 27 November 1978, in revised form 13 February 1979

Abstract. The subject of this paper is a study of interference terms in transition amplitudes of the multiperipheral type. The production amplitude is expressed in the language of continuous-mass-spectrum field theory. The strong-ordering approach is compared with the first term of the correlation expansion. The condition for neglecting correlation terms is found.

So far, to simplify multiperipheral kinematics [1], the strong-ordering hypothesis has been applied [2]. This approximation permits the choice of one multiperipheral graph which dominates over the remaining graphs. Such a factorization of the production amplitude is equivalent to neglecting the correlations among N objects produced in multiperipheral vertices, and it is characteristic of the "short-range correlation" approach to high-energy reactions [2].

The present paper supplements the paper [3], which considered the process of multiperipheral-cluster production. The production amplitude of N objects (clusters) is expressed by means of a field with a continuous mass spectrum [4].

In the present paper, we apply the correlation expansion given by Matsson [5] to the field-theoretic amplitude derived in [3]. This correlation expansion applied to the field-theoretic amplitude allows us to compare the strong-ordering approximation with explicit expressions for correlations of different orders. We conclude that the strong-ordering hypothesis in this field-theoretic formulation is fully equivalent to the first term of the correlation expansion. We give a condition under which one can neglect higher terms in the correlation expansion.

1. Field-Theoretic Approach to Multiperipheral Production Processes

In this section we apply continuous-mass-spectrum field theory [4] to describe the multiperipheral graph shown in Fig. 1. This figure may be considered as a graphical representation of the process pp → pp + N objects. These N objects are produced in N multiperipheral vertices. The objects (of momenta q_1, ..., q_{N+1} and squared masses σ_1, ..., σ_{N+1}) exchanged between multiperipheral vertices and the clusters (of momenta k_1, ..., k_N and masses squared u_1, ..., u_N) produced in vertices are described by a field with a continuous mass spectrum. To each wavy line in Fig. 1 is associated a field with a continuous mass spectrum. In our model all the objects considered are spinless.

A Lagrangian corresponding to this process has the following form:

\[ \mathcal{L}(x) = \frac{1}{2} \int du \rho(u) \left[ \partial_{(n)} \varphi(x, u) \partial_{(n)} \varphi(x, u) - u \varphi(x, u) \right] \]

\[ + \frac{1}{4} \left[ \partial_{(n)} \psi(x) \partial_{(n)} \psi(x) - M^2 \psi(x) \right] + g \psi(x) \Phi^2(x) + G \Phi^4(x), \]

where \( \psi(x) \) is a scalar proton field. The field with continuous mass spectrum in (1) is expressed by

\[ \Phi(x) = \int_{\sigma_0}^{\infty} du \rho(u) \varphi(u, x). \]
In the language of creation and annihilation operators, the field \( \phi(u,x) \) has the following representation:

\[
\phi(x,u) = (2\pi)^{-3/2} \int \frac{d^3k}{2\omega_u(k)} [a^+(k,u)e^{ik.x} + \text{h.c.}],
\]

with commutation relations

\[
\rho(u)\rho(u')[a(k,u),a^+(k',u')] = \rho(u)\delta(u-u')\delta(k-k'),
\]

\[
\rho(u)\rho(u')[a^+(k,u),a^+(k',u')] = \rho(u)\rho(u')[a(k,u),a(k',u')] = 0.
\]

The interaction Lagrangian \( \mathcal{L}_1(x) \) of (1),

\[
\mathcal{L}_1 = g \psi(x) \Phi^2(x) + G \Phi^3(x),
\]

gives vertices as in Fig. 1; \( G \) and \( g \) are the coupling constants for these two types of vertices.

We calculate S-matrix elements according to the rules given in Fig. 2. Neglecting the phase-space factors, we obtain the scattering amplitude for the described reaction in the following form:

\[
M_N = \sum_{\text{perm}} \prod_{i=1}^{N+1} f_i(\{p_1 - p'_i - \mathcal{K}_i\}),
\]

where

\[
f_i(\{\cdot\}) = \int d\sigma_1 \frac{\rho(\sigma_1)}{\sigma_1 - \sigma_i + i\epsilon}.
\]

\[
\mathcal{K}_i = \sum_{r=1}^{i-1} k_r;
\]

or

\[
M_N = \sum_{\text{perm}} \prod_{i=1}^{N+1} d\sigma_i \frac{\rho(\sigma_i)}{\sigma_i - \sigma_j - \epsilon},
\]

where \( \mathcal{K}_i = \sum_{r=1}^{i-1} k_r \).

For further calculations, we choose the mass spectrum function in the form

\[
\rho(\sigma) \sim (\sigma - \sigma_o)^\alpha e^{-\beta\sigma},
\]

where \( \alpha, \beta \) are arbitrary positive constants.

Owing to (5), the integrals in (4) are of the Laplace transform type:

\[
\mathcal{F}_{\sigma_0}[\alpha,\beta,\omega]\ = \int d\sigma_1 \frac{\rho(\sigma_1)}{\sigma_1 - \omega + i\epsilon} e^{-\beta\sigma_1},
\]

where \( \omega = \omega + \sigma_0 \).

The function \( \Gamma(\sigma,\tau) \) is an incomplete gamma function,

\[
\Gamma(\sigma,\tau) = \int e^{-t\sigma} t^{\sigma-1} dt.
\]

[In the denominator of the integral (6) we shall drop the \((-i\epsilon)\) term because the denominator is always positive.]

For \( \sigma = 1 \),

\[
\mathcal{F}_{\sigma_0}[\beta,\omega]\ = \int d\sigma_1 \frac{\rho(\sigma_1)}{\sigma_1 - \omega + i\epsilon} e^{-\beta\sigma_1},
\]

and it is easy to show [3] that

\[
\mathcal{F}_{\sigma_0}(z) = \frac{e^{-\beta\sigma_0}}{\beta} [1 - e^z(z + \sigma_0\beta)E_1(z)],
\]

where \( z = \beta\omega > 0 \), and

\[
E_1(z) = \int e^{-t} \frac{z}{t} dt.
\]

2. Correlation Expansion of Transition Amplitude

In this section we expand the matrix element (2) according to the formula given by Matsson [5].

Using the formula (3) and renumbering indices \( i = 2, \ldots, N+1 \) as \( 1, \ldots, N \), we may rewrite \( M_N \) as follows:

\[
M_N = \int d\sigma_1 \frac{\rho(\sigma_1)}{\sigma_1 - \sigma_2 + i\epsilon} \cdots \rho(\sigma_N) \rho(\sigma) \sum_{\text{perm}} \prod_{i=1}^{N+1} \frac{1}{(p_1 - p'_1 - \mathcal{K}_1)^2 - \sigma_i + i\epsilon},
\]

where \( \omega_i = -t_i = -(p_1 - p'_1 - \mathcal{K}_{i-1})^2 > 0 \) is the absolute value of the squared momentum transfer \( t_i \), for \( i = 1, \ldots, N+1 \).

For further calculations, we choose the mass spectrum function in the form

\[
\rho(\sigma) \sim (\sigma - \sigma_o)^\alpha e^{-\beta\sigma},
\]

where \( \alpha, \beta \) are arbitrary positive constants.

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