Particle production at very high energies in $hh$, $hA$ and $AB$ collisions is studied in the framework of the pomeron model with a strong self-interaction proposed by Cardy. It is shown that the interaction between pomerons substantially damps the production rate one nuclei, the multiplicities for $hA$ collisions becoming $A$-independent.

1 Introduction

Particle production in hadron interactions at high energies is resonably well described by current models of independent string fragmentation [1-3]. They assume that in a collision a certain number of colour strings are formed which independently decay into the observed particles. Although the models differ on the mechanism of the creation of strings as well as on the details of their fragmentation, they all neglect interaction between the strings. The actual number of strings seems to be rather low in $hh$ collisions at present energies, but it is much higher in $hA$ and $AB$ collisions and grows with energy. With many strings in the interaction area there is little reason to assume that they do not interact. Accordingly some study has lately been done to investigate the effects of the interaction between strings [4]. In particular in [5, 6] a simple probabilistic model for these has been proposed. It is shown there that the interaction between strings manifests itself mostly in $hA$ and $AB$ collisions, where it lowers the multiplicities considerably, reducing them in the limit of strong string interactions to those in $hh$ collisions. These results seem to have considerable importance for $hA$ and $AB$ experiments at high energies, as it is worthwhile to study the same phenomena in a more rigorous and less intuitive framework.

From the theoretical point of view the string fragmentation models are certain versions of the old pomeron models in the Regge-Gribov formulation, properly adjusted to finite energies and with a specific choice for the single pomeron fragmentation mechanism. Indeed the main difference between the old pomeron models and the string models consists in a different structure of the vertex for the emission of many pomerons, local in rapidity, and of many strings, nonlocal in rapidity, each hadron constituent emitting strings at its own rapidity. This difference evidently disappears at high enough energies when the rapidity width of the constituent distribution in the hadron becomes much smaller than the typical string rapidity. Because of that it seems reasonable to study the effects of the interaction between strings at high energies taking as a laboratory the old pomeron theory with a self-interaction between pomerons. As far as we know the only pomeron theory general enough that admits explicit study at high energies is a model proposed by Cardy [7]. Accordingly in this note we study the $hh$, $hA$ and $AB$ interactions at high energies in the framework of the Cardy model.

In Sect. 2 we review the relevant results of Cardy in [7]. In his paper Cardy wrongly asserted that particle production in $hh$ collisions is zero in the leading approximation. This is not so and the mistake lies in the well-known AGK cancellations [8], which were not properly taken into account in [7]. This result is presented in Sect. 3, where particle production in $hh$ interactions is studied. Sects. 4 and 5 are dedicated to $hA$ and $AB$ collisions respectively. Some conclusions are made in Sect. 6.

2 Cardy pomeron model

In any pomeron model the $hh$ scattering matrix $S(Y,B)$ at a given impact parameter $B$ and for a given projectile lab. rapidity $Y$ is given as an integral over the pomeron fields $\phi$ and $\phi^{\dagger}$ depending on intermediate rapidities $y$ and impact parameters $b$:

$$S(Y,B)=\int D\phi D\phi^{\dagger} \exp(-a+\alpha_{s}).$$

(1)

The action $a$ in the exponent refers to pomerons proper, the part $\alpha_{s}$ describes their interaction with the colliding...
The hadrons. The pomeron action $a$, in its turn, decomposes in the free part $a_0$ and the pomeron interaction $a_I$. Each of the action is an integral over $y$ and $b$ of the corresponding Lagrangean density, e.g.

$$a = \int_0^y dy \int d^2 b \mathcal{L}(y,b).$$

(2)

The free pomeron Lagrangean is commonly given by

$$\mathcal{L}_0 = (1/2) \phi^* \partial \phi + \alpha' \nabla \phi \nabla \phi - e \phi^* \phi,$$

(3)

with $e$ the intercept minus one and $\alpha'$ the slope of the bare pomeron trajectory. The interaction Lagrangean has a general form

$$\mathcal{L}_I = \sum_{m,n=1} g_{mn} (i \phi)^m (i \phi^*)_n / m! n!$$

(4)

the constants $g_{mn}$ characterizing the coupling of $m$ incoming pomerons to $n$ outgoing ones. The external action has a general structure

$$a_e = \sum_n (h_n (i \phi^*)(Y,B)^n + h'_n (i \phi^*)^n / n!$$

(5)

where the couplings $h_n$ and $h'_n$ refer to the interaction of the projectile and target hadrons with $n$ pomerons respectively.

The formulas written above describe a most general pomeron model with arbitrary self-interaction and coupling to hadrons. The model introduced by Cardy [7] is characterized by the two properties of the couplings

(i) All the couplings $g_{mn}$, $h_n$ and $h'_n$ are analytic in $m$ and $n$, that is admit an analytic continuation in the complex plane $z$ which grows in the right semiplane less rapidly than $\exp (\beta \pi |z|)$ with $0 < \beta < 1$

(ii) The coupling $\varepsilon$ in the free part, which evidently constitutes a nonanalytic addition to the analytic $g_{11}$, is positive

$$\varepsilon > 0.$$  

(6)

This second property gave the name “supercritical” to the Cardy model. Actually it is not exactly so. With the interaction taken account, the intercept becomes $\varepsilon + g_{11}$ even without loops diagrams, which will further renormalize it. It may therefore be positive as well as negative and hardly calculable at moderate energies. Nevertheless we shall see that at asymptotic energies $Y \to \infty$ the Cardy model indeed behaves as if the pomeron intercept were equal to $\varepsilon > 0$. It is supercritical in this restricted sense.

The main results obtained by Cardy are based on a partial summation of multipomeron exchanges between a given pair of vertices to form a “superpropagator” (Fig. 1). Let the first vertex at $y,b$ contain $m_1$ and $n_1$ external lines for incoming and outgoing pomerons respectively and the second one at $0,0$ contain $m_2$ and $n_2$ of these. Then the sum of the diagrams in Fig. 1 is given by

$$G = \sum_{n=1} g_{m_1,n_1+n} \sum_{m_2+n_2} (-p)^n / n!,$$

(7)

where $p$ is the standard pomeron propagator

$$p(y,b) = (4 \pi \alpha' y)^{-1} \exp (e y - b^2 / 4 \alpha' y).$$

(8)

Consider the limit $y \gg 1$. Then the propagator $p$ tends to zero for $b^2 > 4 \alpha' e y^2$ and so the sum $G$ will do. For $b^2 < 4 \alpha' e y^2$ the propagator gets very large. Using the properties of $g_{mn}$ the sum (7) can be converted to the integral

$$G = (1/2i) \int \frac{dn g_{m_1,n_1+n} g_{m_2+n_2}}{(n+1) \sin n \pi}$$

(9)

in the complex $n$ plane with the contour around the points 1, 2, .... In the limit $p \to \infty$ the asymptotical behaviour of the integral (9) is determined by the rightmost singularity on the left of the contour $C$, that is by the pole at $n = 0$. This gives, asymptotically

$$G \approx -g_{m_1,n_1} g_{m_2,n_2} \theta (4 \alpha' e y^2 - b^2), \quad y \gg 1.$$  

(10)

The result (10) means that in the limit $y \gg 1$ the propagators in Fig. 1 sum into the superpropagator

$$d(y,b) = -\theta (4 \alpha' e y^2 - b^2)$$

(11)

coupled to external lines with the constants $g_{m_1,n_1+n}$ and $g_{m_2+n_2}$, where $n$ has to be put equal to zero. To simplify the notation in the following we choose the units in which $4 \alpha' e = 1$, so that (11) becomes

$$d(y,b) = -\theta (y-b).$$

(12)

Performing this summation in all pomeron diagrams we arrive at skeleton diagrams with superpropagators instead of usual propagators, where all internal couplings are equal to $g_{00}$ and all external ones (to the colliding hadrons) are equal to $h_0$ and $h'_0$ for the projectile and target respectively. In the following, for brevity, these will be denoted as $g$, $h$ and $h'$, respectively.

The simple asymptotical form (12) of the superpropagator then leads to the main result obtained by Cardy. It turns out that in many cases addition of a new superpropagator only changes the sign of the contribution of a given diagram, with the consequence that almost all contributions cancel each other (Cardy cancellations). In fact the first of the two diagrams shown in Fig. 2 according to (12) contains the product

$$(-\theta (Y-y - |B-b|)) (-\theta (y-b)).$$

The second one contains an extra superpropagator $-\theta (Y-B)$. However the added $\theta$-function is redundant, since from the first two it already follows that $Y > B$, by the triangle rules.