CORRECTIONS FOR MULTIPLE SCATTERING IN SPHERICAL AND CIRCULAR GEOMETRIES

V. F. Turchin

In this paper a method is considered of introducing corrections for multiple scattering into the results of measurements of angular distributions of elastically scattered neutrons. It is assumed that the mean path of the neutron in the sample in which scattering takes place is comparable with the neutron free path.

In the first part, using direct calculation of the integrals, we find the probability for double scattering and estimate the probabilities for triple and higher-order scattering for a sphere and for a ring of circular and rectangular cross section in the case of isotropic neutron scattering. In the case of anisotropic neutron scattering, at neutron energies of the order of several million electron volts the cross section may be given as a sum \( \sigma(\theta) = \sigma_1(\theta) + \sigma_2(\theta) \), where \( \sigma_1(\theta) \) is the forward peak and \( \sigma_2(\theta) \) is more or less isotropic. Using this representation all elastic scattering events may be provisionally divided into two groups while all double scattering events can be divided into four groups. The probabilities of double scattering for all four are calculated on the basis of results obtained for isotropic scattering. Triple and higher-order scattering are evaluated in similar fashion.

Frequently, in measuring angular distributions for elastically scattered neutrons, scatterers are used whose thickness is comparable with the mean free path of the neutron in the material being investigated; this thickness is used in an attempt to increase the statistical accuracy. In such cases, in analyzing the experimental results it is necessary to take account of the fact that, in addition to singly-scattered neutrons, the detector will register neutrons which have experienced several scattering events. Inasmuch as the dimensions of the scatterer generally will not be large enough so that triple and higher scattering play an important role (if the experiment is to be meaningful), the problem is essentially that of computing the number of doubly-scattered neutrons and estimating the number of neutrons which have been scattered three times or more. This problem is the subject of the present work.

Let the sample being investigated (scatterer) be placed in a neutron flux of unit magnitude.

We introduce the following notation: \( \Sigma_1(\theta, \varphi) \, d\Omega \) is the number of neutrons undergoing single scattering in an element of solid angle \( d\Omega \) in a direction denoted by the angles \( \theta \) and \( \varphi \); \( \Sigma_2(\theta, \varphi) \, d\Omega \) is the same quantity for double scattering; \( \Sigma_3(\theta, \varphi) \, d\Omega = \Sigma_1(\theta, \varphi) \, d\Omega + \Sigma_2(\theta, \varphi) \, d\Omega + \ldots \) applies for triple and higher-order scattering.

It is apparent (Fig. 1), that

\[
\Sigma_1(\theta, \varphi) = \int \sigma(\theta) \, e^{-\mu \rho(\theta) \, d\Omega} \, d\Omega
\]

(1)
where \( \sigma(\phi) \) is the elastic scattering differential cross section, \( \sigma \) is the total cross section, and \( n \) is the number of nuclei of the material per unit volume.

We shall be chiefly interested in the case of circular geometry (Fig. 2). Usually in experiments one measures the ratio \( N/N_0 \), where \( N \) is the number of scattered neutrons recorded by the detector when the direct flux is cut off by a shield while \( N_0 \) is the number of detected neutrons with both the shield and scatterer absent. In an isotropic detector \( N/N_0 \) is related to the macroscopic cross sections as follows:

\[
\frac{N}{N_0} = \frac{R_1^3}{R_1 R_2^3} (\Sigma_1 + \Sigma_2 + \Sigma_m). \tag{3}
\]

**Isotropic Scattering**

First we shall calculate the probability for single and double scattering of neutrons in the case in which \( \sigma(\phi) = \text{const} = \frac{\sigma_s}{4\pi} \) (\( \sigma_s \) is the elastic scattering cross section). Equation (1) is written in the form

\[
\Sigma_e(\theta, \phi) = \int_V n^2 \sigma(\theta) \sigma(\phi) e^{-n\sigma_s (r_1 + r_2)} dV.
\tag{4}
\]

In Equation (2), the factor \( e^{-n\sigma_s (r_1 + r_2)} \) is positive everywhere for double scattering. Its maximum value is unity; the minimum value is not very small inasmuch as we assume that the neutron path in the scatterer does not exceed the free path. Hence, using the mean value theorem and substituting \( \sigma(\phi) = \sigma_s / 4\pi \), Equation (2) may be written as follows:

\[
\Sigma_e(\theta, \phi) = \left( nV \frac{\sigma_s}{4\pi} \right) \frac{1}{V} \int e^{-n\sigma_s (r_1 + r_2)} dV.
\tag{5}
\]