RELATIVISTIC STABILIZED ELECTRON BEAM*

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The physical principles underlying the formation of a relativistic stabilized electron beam are presented and questions connected with the stability of such a configuration are discussed.

1. Introduction

In ordinary accelerators the requirement that \( \text{curl } \mathbf{H} \) and \( \text{div } \mathbf{E} \) vanish in the region of particle motion imposes rather strict limitations on the fields which can be used for acceleration. The removal of this requirement opens new possibilities for building accelerators with extremely strong focusing in which the initial ion ejection is also easier. Thus, for example, it would be possible to accelerate particles in an axially-symmetric magnetic field which increases sharply with radius. In this case both the radial and vertical focusing may prove to be many times greater than that which obtains in ordinary accelerators, including those in which alternate-gradient focusing is employed.

It may be feasible to create such fields by means of a so-called closed stabilized electron beam.

By stabilized electron beam we mean an intense beam of relativistic electrons, the charge of which is completely or partially compensated by ions and which has certain definite properties.

Because of magnetic attraction, the repulsive force between two electrons which move parallel to each other is reduced by a factor \( \gamma^2 \) compared with the repulsion between electrons at rest, where \( \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \) is the relativistic factor. Hence, the presence of even a comparatively small number of ions in an intense relativistic electron beam leads to a situation in which the Coulomb repulsion force vanishes and is replaced by strong attractive force.

Strong magnetic self-focusing causes characteristic electromagnetic radiation which tends to damp out the transverse electron oscillations. If certain other conditions are also satisfied, this effect causes the beam to contract into a thin thread-like structure with enormous electric and magnetic fields at the surface and which is apparently a stable and long-lived configuration.

Since the time typically required for the establishment of a stable configuration (contraction) is of the order of several seconds or more, the electrons traverse enormous distances, and a rectilinear stabilized beam could only be produced in a distance of cosmic dimensions. Under terrestrial laboratory conditions only a closed stabilized beam can be considered. For this reason, the beam must be located in a magnetic field of definite configuration, the direction of which is perpendicular to the plane containing the electron-current loop, as in the betatron.

In contrast with a betatron, however, in the present case the inherent magnetic field at the surface of the beam is much larger than the external magnetic field.

Such a beam is essentially a well-defined, long-lived configuration of electrons and ions which are held together chiefly by the inherent electric and magnetic fields. The failure of many attempts at creating a stabilized classical system, held together only by inherent electromagnetic fields, is well known. Although a rigorous proof of the instability of such a system has never been given, at the present time there is little doubt of the impossibility

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of realizing a stable configuration of this type. A closed stabilized electron beam is not an isolated system inasmuch as it is under the influence of external fields. These fields, which are sufficient for providing stability in the system, are many times smaller than the inherent fields of the beam.

The configuration of the inherent beam field is such that the region inside the beam lends itself to the acceleration of ions. The ions being accelerated are constrained in the orbit and focused by the inherent magnetic and electric field of the beam. If the inherent field is large, one may hopefully expect to obtain very high energy particles with a relatively small accelerator and a small magnetic field, used to constrain the beam to the equilibrium radius; strong focusing and the possibility of using part of the ions which compensate the electron charge for acceleration, may make it feasible to obtain intense beams of accelerated particles, limited only by the power which is available for acceleration.

In the present paper the physical principles which underlie the formation of a stabilized beam are considered and the results of theoretical studies are presented without details or mathematical calculations.

In the near future it is expected that some work by G. I. Budker and S. T. Belyaev on the theory of a relativistic plasma, which has direct bearing on the problem considered here, will be published along with a description of some of the experimental work. The idea that a stabilized electron beam would be possible was advanced by the author in 1952. The basic calculations were carried out in 1953 and experimental work was started in 1954.

The concept of magnetic self-focusing in weak relativistic beams was first advanced by Bennett in 1934 [1]. Recently a second paper by this same author has appeared [2]. Bennett, however, does not consider the radiation produced by the transverse oscillations and it is the existence of this radiation which determines the basic properties of a stabilized electron beam.

2. Equilibrium State

Conditions for mechanical equilibrium. We consider an infinite linear beam of moving electrons the charge of which is partially neutralized by ions. The equilibrium conditions are most obvious if each gas (electron and ion) is considered in the coordinate system in which it is at rest.

If \( n_1 \) and \( n_2 \) are the electron density and the ion density in the laboratory coordinate system, in which the ions are at rest, \( n'_1 \) and \( n'_2 \) are the same densities in the coordinate system, in which the electrons are at rest (electron system), and \( v_0 \) is the directed electron velocity, then from the condition that the current \( j = (n_x; n_y; n_z; n_e) \), a four vector, we must have:

\[
\begin{align*}
\frac{n_1^2 v_0^2}{1} - \frac{n_2^2 v_0^2}{1} &= \frac{n_1^2 c^2}{1} - \frac{n_2^2 c^2}{1} \\
- n_2^2 c^2 &= n_2^2 c^2 - n_2^2 c^2
\end{align*}
\]

where

\[
\begin{align*}
n'_1 &= \frac{1}{\gamma_0} n_1; \\
n'_2 &= \gamma_0 n_2.
\end{align*}
\]

We introduce two dimensionless quantities \( \nu_1 \) and \( \nu_2 \) which will appear frequently in the following*:

\[
\begin{align*}
\nu_1 &= r_0 \cdot \frac{2\pi}{1} \int_0^r n_1 (r) r dr, \\
\nu_2 &= r_0 \cdot \frac{2\pi}{1} \int_0^r n_2 (r) r dr.
\end{align*}
\]

* We may note than when \( \nu_1 \) and \( \nu_2 \) equal unity these values correspond to a beam in which there are \( 3.6 \cdot 10^{12} \) particles per cm, and this in turn corresponds to electron currents (for velocities close to that of light) of approximately 17,000 amp.