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Realization of dimensional reduction at high temperature

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Abstract. Renormalizable four-dimensional field theories reduce at high temperature to effective three-dimensional field models with generically nonlocal interactions induced by the thermal degrees of freedom. Reduction to a local and renormalizable effective model is analyzed here for the example of $SU(N_c)$ lattice gauge theory by means of perturbation theory. The infrared problems are cured by applying the coupled large volume and small coupling expansion. For $SU(2)$ it is shown to the lower orders in this expansion that in the temperature range $T \geqq 3T_c$ dimensional reduction applies, where we consider the following observables: thermal Polyakov line correlations, out of which the interquark potential is derived, and spatial Wilson loops. We also propose an alternative description, in which the effective theory is a gauge theory that lives on a lattice with one time slice and a least number of effective vertices.

1 Introduction

Finite temperature Euclidean field theories exist in a volume with one compact dimension and the appropriate boundary conditions [1–3]. The corresponding radius is given by the inverse temperature T^{-1}. At high temperature T this dimension becomes arbitrary small. The degrees of freedom in T-direction, the nonstatic modes, are essentially frozen. Consider, for instance, the Yang–Mills action

$$S = -\frac{1}{2g^2} \int_0^{T^{-1}} \mathrm{d}x_0 \int \mathrm{d}^3x \sum_{\mu,\nu=0}^{3} \operatorname{tr} F_{\mu\nu}(x) F_{\mu\nu}(x),$$

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) + [A_\mu(x), A_\nu(x)]. \tag{1}$$

For high T this becomes

$$S \cong -\frac{1}{2G^2} \int \mathrm{d}^3x \sum_{\mu,\nu=0}^{3} \operatorname{tr} F_{\mu\nu}(\mathbf{x}) F_{\mu\nu}(\mathbf{x}),$$

with a dimensioned coupling constant $G^2 = g^2 T$. This means that at the classical level the theory reduces to a three-dimensional gauge model that interacts with a scalar field which originates in the T-component of the original Yang–Mills field, and which transforms under the adjoint action of the gauge group. In the general case, as long as we restrict attention to the infrared (IR) properties, a four-dimensional quantum system will behave at high T as a lower dimensional one, with interactions induced by the nonstatic modes and their renormalization.

What does the effective model look like? The corresponding effective action S_{eff} derived by integration of the nonstatic modes normally becomes non-local. An interesting question is whether those modes decouple at high T, which would imply so-called complete dimensional reduction. This has been investigated in detail by Landsman [3] by means of renormalization group arguments. Whether complete dimensional reduction takes place or not depends on the (T-dependent) renormalization scheme and holds only for exceptional field models. A possible candidate is QED [3, 4]. In this case, although there is generation of an electric mass, the magnetic fields stay massless. The electric sector ought to decouple for small momenta $|\mathbf{k}| \lesssim eT$ according to conventional wisdom based on the Appelquist–Carazzone theorem [3, 5–7]. On the other hand, in QCD complete dimensional reduction holds only to some particular order in the gauge coupling g.

An alternative and more general approach is as follows [8–10]. Although the effective model is not a local field theory, the effective action is IR regular. Thus it can be localized by expanding the purely nonstatic parts of the correlation functions about zero momentum. The order of the expansion for every function is determined by the constraint that the nonlocal remnant vanishes in the infinite temperature limit. It is given by three-dimensional power counting and is always finite. Furthermore, it does not depend on the specific order of the diagrams in the coupling constant. The resulting model is a three-dimensional local field theory with effective interactions that are themselves asymptotic series in the coupling constants.

For the example of non-Abelian lattice gauge theory, we show in this paper that the latter approach provides a reasonable description of the full four-dimensional

theory in the distance range $RT \cong 1$ at sufficiently high temperature in the deconfined phase. R denotes the spatial length scale. The organization is as follows. In Sect. 2 we discuss three-dimensional power counting, which determines the order of the effective interactions, and related questions. In Sect. 3 we apply the ideas to the following gauge-invariant quantities: spatial Wilson loops and thermal Polyakov line correlation functions, out of which the interquark potential is derived. A further alternative description that becomes useful beyond perturbation theory, for instance for numerical simulation, is given in Sect. 4, and there also the conclusions are drawn.

2 Effective theory

2.1 Notation

We consider a four-dimensional Yang–Mills theory with gauge group $SU(N_c)$ on a hypercubic lattice Λ of the size $T^{-1} \times \mathcal{L}_s^3 = L_0 a \times (L_s a)^3$ with periodic boundary conditions. a denotes the lattice spacing, and $\mathcal{L}_s$ is the spatial extent of the lattice. The continuum limit is defined by $a \to 0$, $T, \mathcal{L}_s$ fixed, so we always have a finite volume system. The bare action is the standard Wilson plaquette action

$$S_W(U) = \frac{1}{g_0^2} \sum_{\substack{x\in\Lambda \\ \mu,\nu=0,\ldots,4}} \mathrm{Re\,tr}(1 - [U(x;\mu) \cdot U(x+\hat{\mu};\nu)U^{-1}(x+\hat{\nu};\mu)U^{-1}(x;\nu)]), \tag{2}$$

where to each bond $(x;\mu), x = (x_0, \mathbf{x}) \in \Lambda, \mu = 0, \ldots, 3$, a group element $U(x;\mu) \in SU(N_c)$ is associated. The partition function is given by

$$Z = \int \prod_{x,\mu} \mathrm{d}U(x;\mu) \cdot \exp(-S_W(U)), \tag{3}$$

with $\mathrm{d}U$ the Haar measure on $SU(N_c)$. Before we start the perturbative expansion we have to fix the gauge. As the appropriate gauge in the thermal situation we choose the static time averaged Landau gauge (STALG) [11, 12]. As pointed out in [13], even in finite volume this allows us to restrict attention to a neighborhood Ω of all $U(x;\mu)$ equal to 1. We parametrize Ω by

$$U(x;\mu) = \exp aA_\mu(x), \quad x \in \Lambda, \mu = 0, \ldots, 3, \tag{4}$$

where the Lie algebra valued gauge field A is subject to the constraints

$$\frac{1}{L_0} \sum_{x_0} \sum_{i=1}^{3} \partial_i^* A_i(x_0, \mathbf{x}) \equiv \frac{1}{L_0 a} \sum_{x_0} \sum_{i=1}^{3} (A_i(x_0, \mathbf{x}) - A_i(x_0, \mathbf{x} - \hat{\imath})) = 0, \tag{5}$$

and

$$A_0(x_0, \mathbf{x}) \equiv A_0(\mathbf{x}). \tag{6}$$

We use the representation

$$A_\mu(x) = \sum_{d=1}^{N_c^2-1} A_\mu^d(x) T^d$$

with anti-hermitian traceless T, which are normalized according to

$$\mathrm{tr}\, T^d T^e = -\tfrac{1}{2}\delta_{d_e}.$$

The zero momentum modes are ultimately separated off by

$$A_\mu(x) = g_0 W_\mu(x) + B_\mu, \quad \sum_{x\in\Lambda} W_\mu(x) = 0. \tag{7}$$

The partition function assumes the form

$$Z \cong \int \prod_{x,\mu} \mathrm{d}A_\mu(x) \cdot \exp(-S_0(A)). \tag{8}$$

The full action S_0 can be found in [12]. We do not make it explicit here. Those Feynman rules which are particular to STALG are given in [13].

We write the gauge field A as the sum of static and nonstatic modes,

$$A_\mu(x) = T^{1/2} A_\mu^{\mathrm{st}}(\mathbf{x}) + A_\mu^{\mathrm{ns}}(x), \quad \sum_{x_0} A_\mu^{\mathrm{ns}}(x) = 0. \tag{9}$$

The reason for the scaling of the static modes with a factor $T^{1/2}$ will become clear below. For the Fourier transformed fields

$$\tilde{A}_\mu(k) = a^4 \sum_x \exp(-\mathrm{i}k\cdot x) \exp\left(-\mathrm{i}k_\mu \frac{a}{2}\right) A_\mu(x).$$

This implies

$$\tilde{A}_\mu(k) = T^{-1/2} \tilde{A}_\mu^{\mathrm{st}}(\mathbf{k}) \delta_{k_0,0} + \tilde{A}_\mu^{\mathrm{ns}}(k)(1 - \delta_{k_0,0}), \tag{10}$$

with

$$\tilde{A}_\mu^{\mathrm{st}}(\mathbf{k}) = a^3 \sum_{\mathbf{x}} \exp(-\mathrm{i}\mathbf{k}\cdot\mathbf{x}) \exp\left(-\mathrm{i}k_\mu \frac{a}{2}\right) A_\mu^{\mathrm{st}}(\mathbf{x}),$$

$$\tilde{A}_\mu^{\mathrm{ns}}(k) = a^4 \sum_x \exp(-\mathrm{i}k\cdot x) \exp\left(-\mathrm{i}k_\mu \frac{a}{2}\right) A_\mu^{\mathrm{ns}}(x).$$

The partition function can now be written as

$$Z = Z_{\mathrm{st}} \cong \int \prod_{\mathbf{x},\mu} \mathrm{d}A_\mu^{\mathrm{st}}(\mathbf{x}) \cdot \exp(-S_{\mathrm{eff}}(A^{\mathrm{st}})), \tag{11}$$

where the effective action S_{eff} is derived according to

$$\exp(-S_{\mathrm{eff}}(A^{\mathrm{st}})) = Z_{\mathrm{ns}}(A^{\mathrm{st}}) = \int \prod_{x,\mu} \mathrm{d}A_\mu^{ns}(x) \cdot \exp(-S_0(T^{1/2}A^{\mathrm{st}} + A^{\mathrm{ns}})). \tag{12}$$

The expectation value of any gauge invariant quantity $I(U)$ becomes

$$\langle I \rangle \cong \frac{1}{Z_{\mathrm{st}}} \int \prod_{\mathbf{x},\mu} \mathrm{d}A_\mu^{\mathrm{st}}(\mathbf{x}) \cdot \exp(-S_{\mathrm{eff}}(A^{\mathrm{st}})) I^{\mathrm{st}}(A^{\mathrm{st}}), \tag{13}$$

with

$$I^{\mathrm{st}}(A^{\mathrm{st}}) = \frac{1}{Z^{\mathrm{ns}}(A^{\mathrm{st}})} \int \prod_{x,\mu} \mathrm{d}A_\mu^{\mathrm{ns}}(x) \cdot \exp(-S_0(T^{1/2}A^{\mathrm{st}} + A^{\mathrm{ns}})) I(T^{1/2}A^{\mathrm{st}} + A^{\mathrm{ns}}). \tag{14}$$

2.2 Power counting

In general, the effective action S_{eff} is non-local. But, for high temperature it becomes more and more local in the