Diese Bedingung ist um so besser erfüllt, je mehr sich die Flüssigkeit einem wirklichen Kontinuum nähert. Wirkliche Abweichungen kommen erst dann zustande, wenn man grobe Suspensionen hat — also nicht eine Flüssigkeit, sondern Körnungen, die aneinander vorbei strömen. Wenn das nicht so wäre, könnte man z. B. an Quecksilber, das an Glas gar nicht haftet, keine Viskositätsmessungen durchführen. Aber da die Kontinuumsbedingung erfüllt ist, da Quecksilber eine hochdisperse Flüssigkeit ist und die Teilchen im Verhältnis zu den Kapillardimensionen außerordentlich klein sind, kann man von Quecksilber sehr genaue Viskositätsmessungen machen. Es muß also immer die Spaltbreite groß sein gegenüber den Molekülen.

Zeidler (Berlin): Sie streben eine geringe Spaltbreite. Spielt denn nicht der Temperaturfluß, der sich durch die Reibung ergeben kann, eine erheblichere Rolle als bei einem breiteren Spalt, trotz der Temperiervorrichtung?

Groß (Celle): Die erzeugte Wärmemenge hängt wohl lediglich ab von der Scherkraft, mit der ich das System bewege.

Schreck (Berlin): Wir haben erwogen, etwa folgendes zu bauen: den inneren, sich drehenden Zylinder so auszuführen, daß man unten eine Luftblase einschließt. Dadurch hätte man die Reibung am Boden vermieden.

Umstätter (Berlin-Dahlem): Um diese Einflüsse auszuschalten, macht man die Füllhöhe verschieden groß, trägt die Unterschiede auf und extrapoliert dann. Die Unterschiede sind um so größer, je größer der Unterschied von Ringfläche gegenüber der Zylinderfläche ist.

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A Theory of Rheodynamic Lubrication*)
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With 5 figures
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1. Introduction

Reynolds' equation for pressure gradient in hydrodynamic lubrication was obtained on the assumption, inter alia, that the viscosity of the lubricant remained very nearly constant throughout the film. If the viscosity of the lubricant changes slowly along the length of the bearing due e.g. to temperature rise, then the equation is still applicable at each section. But if, as Cameron (1) has pointed out, the viscosity varies across the lubricant film due to e.g. a temperature gradient, then Reynolds’ equation is no longer valid and the expression for friction and pressure must be re-derived. This applies when lubricants, that are not-Newtonian fluids, are used in fluid film bearings; as the shear stress varies across the film so does the local viscosity. The change in viscosity is likely to be particularly marked with a plastic lubricant such as grease, but a similar effect will be noted with structurally viscous oils. Both types of lubricant have been shown experimentally (2, 3) to behave in a manner slightly different from that of normal oils. It is the purpose of this note to suggest a theoretical treatment for determining the behaviour of such viscous lubricants that are not Newtonian, with particular reference to greases. As the theory is sufficiently general to include plastics as well as fluids it is considered that the term ‘hydrodynamic lubrication’ is inappropriate, and it is proposed to use instead the description ‘rheodynamic lubrication’.

2. A Theory of Rheodynamic Lubrication

For the purpose of this note it will be sufficient to consider only the behaviour of an infinitely long bearing, i.e. neglecting side leakage. The usual assumptions of hydrodynamic lubrication will be made, save only that the viscosity is no longer regarded as a constant. Under the conditions considered the basic equation of flow may be shown (4) to be:

\[
\frac{\partial p}{\partial x} = \frac{\partial x}{\partial y}
\]

and hence

\[
\tau = y \cdot \frac{\partial p}{\partial x} + c_1
\]

If the flow properties of the lubricant are expressed in the general form

\[
\frac{\partial u}{\partial y} = F(r)
\]

then by substitution and integration
\[ \frac{\partial u}{\partial y} = F \left[ y \frac{\partial p}{\partial x} + c_1 \right], \quad [4] \]

\[ u = \int_0^y \int_0^x F \left[ y \frac{\partial p}{\partial x} + c_1 \right] dy + c_2, \quad [5] \]

\[ Q = \int_0^h \int_0^b F \left[ y \frac{\partial p}{\partial x} + c_1 \right] dy + c_2, \quad [6] \]

where \( c_1 \) and \( c_2 \) are constants depending on the velocities of the bearing surfaces and \( Q \), the volume flow per unit width, is constant across all sections of the bearing.

In the general case, where only a graphical form of equation [3] is obtainable, it would be necessary to employ graphical methods for integration of equations [5] and [6] to obtain the required variable \( \frac{\partial p}{\partial x} \). Such a solution, while possibly tedious, is quite straightforward. For the simplest case of a Newtonian fluid of viscosity \( \eta \), on the other hand, where

\[ F[\tau] = \frac{\tau}{\eta}, \quad [7] \]

then equation [1] becomes

\[ \frac{\partial p}{\partial x} = \eta \frac{\partial u}{\partial y^2}, \quad [8] \]

and for the boundary conditions \( y = 0, u = -U; y = h, u = 0 \); equation [6] reduces to

\[ Q = -\frac{1}{12 \eta h^2} \frac{\partial p}{\partial x} = \frac{Uh}{2}, \quad [9] \]

which is a form of the usual equation for the pressure gradient in a hydrodynamic bearing lubricated with a Newtonian fluid.

If the conditions of operation of the bearing are such that the variation of shear stress across the film is small enough for the relevant portion of the flow curve to be approximately linear, i.e. such that

\[ F[\tau] \approx \frac{\tau}{\eta} + k \quad [10] \]

then equations [8] and [9] are again obtained. Note, however, that the effective viscosity governing pressure generation is not the absolute viscosity based on the mean rate of shear, i.e.

\[ \mu = \frac{\tau}{\left( \frac{\partial u}{\partial y} \right)} = \frac{\tau}{\left( \frac{U}{h} \right)}, \quad [11] \]

but the mean slope, \( \eta \), of the flow curve at this point. This is essentially the approximation made by Umstätter (3), but it is valid only under the limited conditions mentioned.

When there is appreciable curvature of the flow curve neither the viscosity based on a mean rate of shear across the film nor the mean slope of the flow curve are applicable, and special consideration is required for each particular case. It may sometimes be possible to obtain analytical solutions for equation [6] at different sections along the bearing by fitting simple integrable expressions to the relevant portions of the flow curve.

3. Grease in Rheodynamie Lubrication

Greases form a particularly interesting field for study in the theory of rheodynamic lubrication, for their flow properties may be idealised as those of Bingham plastics, i.e.

\[ \frac{\partial u}{\partial y} = \frac{1}{\eta} \left( \left| \frac{\tau}{\eta} - \tau \right| \right) \quad [12] \]

In those sections of the lubricant film where the shear stress always exceeds the yield value, \( \tau_0 \), the behaviour of grease is similar to that of other materials with a linear portion of the flow curve and equations [8] and [9] are again obtained. In other sections of the film where the shear stress in places does not exceed the yield value it is necessary to integrate equation [6] through part or all of the discontinuity of equation [12]. The resultant expressions for the pressure gradient may then be applied to specific types of bearing. Figure 1 shows a typical pressure curve for a slider bearing and figure 2 shows the calculated load capacity. For comparison, the curves for

\[ p = \frac{(\alpha - 1) h^2}{2 \eta L} \quad \text{By hydrodynamic theory} \]

\[ p = \frac{(\alpha - 1) h^2}{2 \eta L} \quad \text{By rheodynamic theory} \]

Figure 1. The variation of pressure along a slider bearing; a comparison, for an arbitrary maximum/minimum film thickness ratio, between a Bingham plastic and a Newtonian fluid of the same effective viscosity \( \eta \).