Anderson Localization in Different One-Dimensional Systems with Off-Diagonal Disorder and Spin-Dependence

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Received October 6, 1983

The localization properties of certain spin-dependent one-dimensional electronic systems with only off-diagonal disorder are studied. In higher dimensions \( (d = 2, 3) \) the models considered would correspond to different universality classes, whereas for \( d = 1 \) no qualitative difference is found: For \( E = 0 \) all eigenstates are exponentially localized, whereas for \( E \to 0 \) the localization length diverges logarithmically, such that exactly at \( E = 0 \) the geometric average of the transmission coefficient would decay with increasing chain length \( L \) as \( \exp (-\text{const} \cdot L^{1/2}) \), instead of the usual exponential decay.

For \( E = 0 \), in the interior of the band, the localization length \( r_0 \) diverges \( \sim W_2^{-2} \) in the limit of weak disorder \( (W_2 \to 0) \), whereas just at the band edge one has roughly \( r_0 \sim W_2^{-2/3} \). A universal recursion relation, depending only on the energy and on certain randomly distributed determinants, determines the localization length and the density of states for all systems considered.

1. Introduction

Are there different universality classes for the Anderson localization problem? This question has attracted a lot of interest in recent years [1–3], particularly for two-dimensional systems \( (d = 2) \). Analytically, by perturbational methods [4], \( 1/n \)-expansions [5], \( (2 + \epsilon) \)-expansions for a field-theoretic version of the problem [6, 7], and an algebraic derivation using a supersymmetric field theory [8], it has been found that there are (at least) three different universality classes with quite distinct behaviour, namely \( (i) \) the orthogonal one, corresponding to the case of potential scattering; \( (ii) \) the symplectic one, corresponding to the simultaneous presence of spin orbit and potential scattering; and \( (iii) \) the unitary one, where in contrast to the two first-mentioned cases time-reversal invariance is broken, due to scattering by magnetic impurities. According to what is known from [2–8], in the weak-scattering limit for \( d = 2 \) these models behave quite differently, giving rise to the phenomenon of weak localization for case \( (i) \), weak antilocalization in case \( (ii) \), and extraordinarily-weak localization in case \( (iii) \); More details can be found in a recent paper [9], where also some numerical evidence has been presented that in case \( (i) \) and \( (iii) \) there is no sharp transition, but only a crossover between weak and strong localization, if the Fermi energy changes from the interior of the band to the band tail, for given disorder; whereas in the symplectic case, \( (iii) \), according to [9] there seems to exist a transition with a critical energy separating a phase corresponding to weak antilocalization (i.e. where the ac-conductivity would increase logarithmically with decreasing frequency) for energies in the interior of the band, from a phase corresponding to exponentially localized states in the band tails. Thus for \( d = 2 \) there is a drastic difference between the above-mentioned three universality classes.

Now, according to [5], these universality classes are represented by relatively simple lattice Hamiltonians, which are well-defined also for the one-dimensional case (see below!). Therefore it would be interesting to know whether these different Hamiltonians show qualitatively different localization behaviour even for \( d = 1 \), or whether in that case all states would be exponentially localized for all three classes, without any qualitative difference. This question is pursued by the present study, mainly numerically and by some additional analytical arguments. The models considered contain only off-diagonal nearest neighbour-
hopping, whereas the diagonal matrix elements vanish identically.

As a result it is found that in all cases the states of the system should be exponentially localized, even for the model corresponding to the above-mentioned symplectic case. Moreover, the particular gauge-invariant representations of the above-mentioned symplectic and unitary cases (see below) have identical values of the localization length as a function of the energy for given strength of disorder, while, on the other hand, for the gauge-invariant version of the orthogonal case, which has already been studied in \[10-12\], the localization lengths are shorter. Finally in all cases, due to the constancy of the diagonal matrix elements, the states at the band center (i.e. \( E = 0 \)) show a particular behaviour: There the localization length diverges logarithmically (i.e. \( \sim \ln (E^{-2}) \)), for all cases considered, and the geometric mean of the transmission coefficient for \( E = 0 \) does not vanish exponentially but as \( \exp (-\text{const.} \cdot E^{1/2}) \). For \( E \neq 0 \) on the other hand, in the limit of weak disorder (\( W^2 \to 0 \)) the localization length \( r_0 \) is always \( \sim W^{-2} \) as long as \( E \) would lie in the interior of the band for \( W^2 = 0 \), whereas at the band edges one has roughly \( r_0 \sim W^{-2/3} \). For the orthogonal case these results are already known from \[10-12\], except of the last statement.

After all it turns out that the different models considered behave universal in the following sense: The localization length depends only on a certain recursion relation, (\( B1 \)) of Appendix B, which does not distinguish explicitly between the cases considered, and which depends only on the energy and certain randomly distributed determinants.

In the following Chap. 2 the models and basic definitions are introduced: then in Chap. 3 the numerical method for the determination of the localization length is explained; Chapter 4 presents the results, and Chap. 5 the conclusions. Finally in Appendix A the behaviour at \( E = 0 \) is treated analytically, and in Appendix B the above-mentioned universal recursion relation is derived, from which the logarithmic divergence for \( E \to 0 \) and the behaviour for \( W^2 \to 0 \) follow.

2. Model Definitions

The Hamiltonians considered in the following are of the tight-binding type

\[
H = \sum_{l,m} \sum_{\sigma, \sigma'} H_{l,m}^{\sigma, \sigma'} |l, \sigma \rangle \langle m, \sigma'|. \tag{1}
\]

Here \( l \) and \( m \) denote the sites (of an infinite chain, in the present case); \( \sigma \) and \( \sigma' (\pm 1) \) are spin indices, and \( |l, \sigma \rangle \) and \( |m, \sigma' \rangle \), respectively denote the orthonormal basis of the system, i.e. it is

\[
\langle l, \sigma | m, \sigma' \rangle = \delta_{l, m} \delta_{\sigma, \sigma'}. \tag{2}
\]

In the following it is always assumed that the site-diagonal matrix elements \( H_{l,m}^{\sigma, \sigma} \) vanish; additionally only nearest-neighbour off-diagonal matrix elements, i.e. \( H_{l,m}^{\sigma, \sigma'} \) with \( l - m = \pm 1 \), are assumed to be different from zero. Furthermore it is assumed that the Hamiltonian not only fulfills the usual condition of hermiticity

\[
H_{l,m}^{\sigma, \sigma'} = (H_{l,m}^{\sigma', \sigma})^* \tag{3}
\]

but additionally (and this is important for the following) the condition of invariance against time reversal

\[
H_{l,m}^{\sigma, \sigma'} = \sigma \cdot \sigma' (H_{l,m}^{\sigma, \sigma'})^*. \tag{4}
\]

Thus for a given pair of neighbouring sites \( (l, m) \) only two matrix elements can be freely chosen, e.g. \( H_{l,m}^{+, +} \) and \( H_{l,m}^{-, -} \), while the rest follows from (3) and (4).

The three different universality classes mentioned above, see \[5\], would then correspond to the following specifications for the hopping matrix-elements \( H_{l,m}^{\sigma, \sigma'} \):

(i) In the orthogonal case one assumes that there is only potential scattering, i.e. \( H_{l,m}^{\sigma, \sigma'} = 0 \), and no spin dependence of the potential, i.e. \( H_{l,m}^{+, +} = H_{l,m}^{-, -} \). Then by (3) and (4) the spin indices can be omitted as dummy variables, and the matrix elements \( H_{l,m} \) are necessarily real and symmetric: \( H_{l,m} = (H_{l,m})^* = H_{m,l} \). Specifically we assume that the nonvanishing elements \( H_{l,l+1} \) are drawn independently from a rectangular distribution with average \( \langle H_{l,l+1} \rangle = V \) and width \( W_2 \). For \( V = 0 \) this would yield a particular local gauge-symmetry, namely all ensemble averages would be invariant against changes of the sign of the wave function at a given site \( l \), which leads to an enormous simplification in the perturbation theory, as exploited extensively in \[13\] (e.g. all single-particle averages are site-diagonal).

Although it has been shown that the models with \( V = 0 \) and \( V \neq 0 \) belong to the same universality class, \[14\], both cases will be considered below: The reason is the following: For \( V = 0 \) the only energy scale is given by \( W_2 \), which characterizes the strength of the disorder, and the results depend only on the reduced energy \( E/W_2 \); therefore, if one wants to study more generally both the influence of the energy and of the disorder on the localization length, one must assume \( V \neq 0 \), to obtain results depending both on a reduced energy \( E/V \) and a reduced disorder \( W_2/V \).

(ii) In the so-called symplectic case the spins are present, and the \( H_{l,m}^{\sigma, \sigma'} \) are complex. It is assumed that the independent matrix elements \( H_{l,l+1}^{+, +} \) and \( H_{l,l+1}^{-, -} \) are given by

\[
H_{l,l+1}^{\sigma, \sigma'} = V \cdot \delta_{\sigma, \sigma'} + (H')^{\sigma, \sigma'}_{l,l+1} \tag{5}
\]

where \( V \) is real, whereas \( (H')^{+, +}_{l,l+1} \) and \( (H')^{-, -}_{l,l+1} \) are two independent complex random numbers: The magni-