The stick-slip problem for a round jet
II. Small surface tension

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With 18 figures

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Summary

The stick-slip problem for a round jet studied in Part I gives a good approximation for the swell of a low speed jet when the surface tension is large but it fails when the surface tension is small. In this paper a new stick-slip problem (II) is defined and solved using matched eigenfunction expansions. The new problem reduces to that solved in Part I when the surface tension is large and gives good results in the case of zero and small surface tension.

Zusammenfassung

Das in Teil I untersuchte Haft-Gleit-Problem für einen runden Strahl liefert eine gute Näherung für die Strahlaufweitung bei langsamer Austrittsgeschwindigkeit, wenn die Oberflächenspannung groß ist, versagt dagegen bei kleiner Oberflächenspannung. In der vorliegenden Veröffentlichung wird daher ein neues Haft-Gleit-Problem (II) definiert und mittels aneinander angeschlossener Eigenfunktionsentwicklungen gelöst. Dieses Problem geht in das schon früher in Teil I gelöste Problem über, wenn die Oberflächenspannung groß ist, liefert aber auch bei kleiner oder gar verschwindender Oberflächenspannung gute Ergebnisse.

Key words

Stick-slip, low speed jet, surface tension, matched eigenfunction expansion
and is increasingly inaccurate when the surface
tension is small. In this paper we define a new
stick-slip problem which can be solved by eigen-
function methods for all values of surface ten-
sion including zero. The problem reduces to
that treated in Part I when the surface tension is
infinite and is in good agreement with direct
numerical computations for zero and small sur-
fase tension.

2. Governing equations

A Newtonian fluid is extruded from a semi-
finite ($\tilde{x} < 0$) circular pipe of radius $\tilde{r} = a$ by an
applied uniform pressure gradient as $\tilde{x} \to -\infty$
giving rise to Poiseuille flow there. A jet of fluid
of radius $\tilde{r} = \tilde{h}(\tilde{x})$, $\tilde{h}(0) = a$, $\tilde{x} \geq 0$ is extruded
from the pipe. It is assumed that wind shears
and body forces are negligible so that the shear
stress on $\tilde{r} = \tilde{h}(\tilde{x})$ vanishes and the normal
stresses are balanced by interfacial tension.
Under such assumptions the flow far upstream,
$\tilde{x} \to \infty$ is constant with final velocity
$\vec{u}_{\infty} = \vec{U}_{f}$ and jet radius $\tilde{h} = \tilde{h}_{f}$. The swelling ratio is
defined as $\chi = \tilde{h}_{f}/a$ (see fig. 2.1).

With inertial effects neglected the Stokes
equations of motion are

$$\nabla \mathbf{T} = \text{div} \mathbf{u} = 0, \quad [2.1]$$

where

$$\mathbf{T} = -\tilde{\phi} I + \tilde{\mathbf{S}}$$

is the total stress,

$$\mathbf{S} = \mu \left[ \text{grad} \, \mathbf{u} + (\text{grad} \, \mathbf{u})^\top \right]$$

is the deviatoric stress and

$$\tilde{\mathbf{S}} = \tilde{\mathbf{S}} + \tilde{\mathbf{P}} - \tilde{\mathbf{P}} \tilde{a}$$

is the reduced pressure for zero body force. Eq.
[2.1], along with the requirement that the flow
be axisymmetric, implies the existence of a
stream function $\Psi(\tilde{x}, \tilde{r})$. We are interested in
determining this stream function and the jet
shape $\tilde{h}(\tilde{x})$ for different values of surface ten-
sion, $\sigma$. We seek these field quantities in terms
of dimensionless variables

$$\tilde{x}, \tilde{r} = \frac{1}{a} (\tilde{x}, \tilde{r})$$

$$\Psi(\tilde{x}, \tilde{r}) = \tilde{\Psi}(\tilde{x}, \tilde{r})/\varepsilon,$$

where

$$\varepsilon = \frac{\int_0^{\tilde{h}(\tilde{x})} \mathbf{e}_x \cdot \mathbf{u}(\tilde{x}, \tilde{r}) d\tilde{r}}{2\pi}$$

is independent of $\tilde{x}$,

$$\mathbf{u} = \frac{\bar{\mathbf{a}}^2}{\varepsilon} (\tilde{u}, \tilde{w}) = \frac{a^2}{\varepsilon} (\bar{u}, \bar{w}),$$

$$\mathbf{S} = \frac{a^3}{\varepsilon} \tilde{\mathbf{S}},$$

$$\tilde{\phi} = \frac{a^3}{\varepsilon} \tilde{\phi}$$

and

$$h(\tilde{x}) = 1 + \eta(\tilde{x}) = \tilde{h}(\tilde{x})/a.$$