STRESS STATE IN NONAXISYMMETRIC DISKS

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First we will look at the determination of the internal stresses in a solid elliptical disk. Finding the internal stresses in such a disk requires that the cross section be conformally mapped onto a unit circle [1, 2]. We will use the opposite approach and conformally map the axisymmetric and (we assume) linear distribution of the internal stress field for a circular disk onto a solid elliptical disk.

We dissect the ellipse (Fig. 1a) along its major axis and examine only the top part of the half-ellipse (Fig. 1b). We perform a similitude transformation, having mapped the top half-ellipse onto a half-ring:

\[ W_1(z) = \frac{1}{c}(z + \sqrt{a^2 - b^2}) \tag{1} \]

where \( c = \sqrt{a^2 - b^2} \) is the linear eccentricity of the ellipse; \( z \) is a variable.

We use Eq. (1) to find the position of the points \( a' \) and \( b' \) on the half-ring.

The pole points will be found at distances \( +1 \) and \( -1 \) from the center of the major axis.

We map the half-circle onto the plane of a rectangle (Fig. 1c):

\[ W_2(z) = \ln W_1 \tag{2} \]

so that the point \( a' \) is converted to the image of the point \( a', \) at a distance \( \ln a \) from the center.

We rotate the mapped rectangle by 90° (Fig. 1d):

\[ W_3(z) = \frac{1}{\ln a} z + \frac{\pi}{2\ln a} + i \tag{3} \]

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Fig. 1. Mapping of a solid elliptic disk onto a circle.

We use the elliptic sine function to map the rotated rectangle onto the half-plane

\[ W_4(z) = \text{sn}(W_3, k) = \frac{z}{\sqrt{1 - k^2 z^2}}, \]

where \( k \) is the ratio of the sides of the rectangle.

We convert the half-plane of the region \( W_4 \) into a half-plane of the region \( W_5 \):

\[ W_5(z) = \frac{W_4 + 1}{W_4 + i}. \]

We map the half-plane \( W_4 \) onto the one-quarter-plane:

\[ W_6(z) = \sqrt{W_5}. \]

We map the region \( W_6 \) onto a unit half-circle (Fig. 1e).

Based on the principle of symmetry, \( W_7 \) will be the sought mapping on the unit circle:

\[ W_7(z) = \frac{W_6 - i}{-W_6 - i} = 1. \]

The Zhukov function is used to obtain the actual value of the radius of the circle:

\[ W(z) = \frac{1}{2} \left(z + \frac{1}{z}\right). \]

Having taken \( z = \lambda e^{i\varphi} \) in Eq. (8) with allowance for \( \lambda^2 = x^2 + y^2 \) and using Euler's formulas

\[ e^{i\varphi} = \cos \varphi + i \sin \varphi; \]

\[ e^{-i\varphi} = \cos \varphi - i \sin \varphi, \]

we find:

\[ W(z) = \frac{1}{2} \left( \lambda (\cos \varphi + i \sin \varphi) + \frac{1}{\lambda} (\cos \varphi - i \sin \varphi) \right). \]