ACCURACY OF PREDICTION OF THE ENDURANCE OF STRUCTURAL MEMBERS IN CREEP CONDITIONS ON THE BASIS OF CRACKING RESISTANCE CHARACTERISTICS OF THE MATERIAL

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In applying fracture mechanics in practice for predicting the endurance of structural members working in creep conditions, it is necessary to determine the accuracy of the prediction method. It is obvious that this problem must be urgently solved because it is linked with the solution of the general problem of strength reliability.

The problems of the accuracy of prediction of the endurance of structural members on the basis of the cracking resistance characteristics of the material in the area of application of fracture mechanics to creep processes in practice have not been reflected to a sufficient extent in the studies carried out by investigators.

The aim of this report is to determine the effect of procedure errors in the experimental determination of the cracking resistance characteristics of the material on the accuracy of prediction of the endurance of structural members in creep using the criteria of linear fracture mechanics.

In the general case, the accuracy of prediction of endurance or relative error of the method based on the application of fracture mechanics is determined by the set of errors caused by the experimental method of testing the cracking resistance of materials in the laboratory conditions, by the method of mathematical processing of the test results, and also by mathematical procedures used in calculating the endurance of the structural member from the cracking resistance characteristics of the material.

Calculations of the endurance of structural members on the basis of the cracking resistance characteristics of the material within the framework of linear fracture mechanics are carried out using the kinetic equation of subcritical growth of the creep crack in the form [1]

\[ \ell = AK_0^n, \]  

where \( \ell = \frac{d\ell}{dt} \) is the rate of subcritical growth of the creep crack; \( K_1 \) is the stress intensity factor (SIF); \( A, n \) are the parameters of the equation which depend on the properties of the material.

Equation (1) is invariant for the structural member and the specimen made from the material of the given member. The endurance or crack growth time in the structural member from initial length \( L_0 \) to final length \( L_f \) (maximum permissible) is calculated by integrating Eq. (1)

\[ t = \frac{L_f}{A} \int_{L_0}^{L_f} K_1^{-a} dl. \]  

The parameters of Eq. (1), \( A, n \), are determined in the course of creep testing standard crack-containing specimens in the laboratory conditions. The SIF is determined by numerical calculation methods. The initial and running crack dimensions in Eq. (2) are measured in the structural member. To simplify Eq. (2), in subsequent considerations and analysis we shall examine the averaged-out value of SIF \( (K_1^*) \) in the form

\[ K_1^* = \left[ \frac{L_f}{L_0} \right] K_1(l) dl/(L_f - L_0). \]  

Substituting Eq. (3) into (2) and integrating, we obtain
The difference between the endurance values obtained from Eqs. (4) and (2) is characterized by the constant multiplier $\xi$

$$t = \xi t^*.$$  \hspace{1cm} (5)

We shall show that the relative errors for the averaged-out endurance $t^*$ and endurance $t$ are identical. We shall use the following relationships [2] for this purpose

$$\delta_t = \frac{t - \bar{t}}{\bar{t}}; \quad \delta_{t^*} = \frac{t^* - \bar{t}^*}{\bar{t}^*}.$$  \hspace{1cm} (6)

where $\bar{t}$, $\bar{t}^*$ are the mathematical expectations of the corresponding endurances; $\delta_t$, $\delta_{t^*}$ are the relative errors of the endurances.

Expressing in Eq. (5) the running values of the endurances by means of the relative errors and mathematical expectation, and carrying out simple transformations, we obtain

$$\delta_t = \delta_t^*.$$  \hspace{1cm} (7)

Equality (7) shows that in subsequent considerations in determining the relative error we can use Eq. (4) which is simpler and more descriptive.

To determine the relative error of the predicted endurance, we shall use Eq. (4) together with the equation from the general error theory [2]. After simple transformations, we obtain

$$\delta_t = n \ln K^* \delta_n + n \delta_{K^*} + \delta_\lambda + \delta_f.$$  \hspace{1cm} (8)

where $\delta_t$ is the relative error of the predicted endurance of the structural member; $\delta_\lambda$, $\delta_f$ are the relative errors of the parameters of the material $A$, $n$ caused by the theoretical and experimental method of their determination; $\delta_{K^*}$ is the relative error of the averaged-out SIF determined for the structural member; $\delta_f$ is the relative error associated with the accuracy of measurement of crack length in the structural member.

Equation (8) holds in the case in which the components of the relative errors fulfill the following conditions

$$|\delta_n| \ll 1; \quad |\delta_{\lambda}| \ll 1; \quad |\delta_K^*| \ll 1; \quad |\delta_f| \ll 1.$$  \hspace{1cm} (9)

At $n > 1$ and $\ln K^* > 1$, Eq. (8) indicates that the largest contribution to the total error of the predicted endurance comes from parameter $n$ since its sensitivity, expressed by the absolute value of the cofactor at this parameter ($n \ln K^*$), is the highest in Eq. (2) in comparison with other values. This error is followed by the relative error of the method of determining the SIF in the structural member; its sensitivity is determined by multiplier $n$ in Eq. (8). The contribution of the relative errors of parameter $A$ and the crack length to the total error of endurance (8) is identical since their sensitivity is equal to unity and is the lowest in comparison with the sensitivity of the previously discussed errors.

To derive the equation for determining the relative error of the method of determining SIF, we shall examine the mathematical expression [1] used to write SIF in the form

$$K_\gamma = \sigma_\infty \sqrt{f(\lambda)},$$  \hspace{1cm} (9)

where $\sigma_\infty$ is the nominal stress acting in the structural member; $\lambda$ is the length of the crack in the structural member; $\lambda$ is the crack length (dimensionless) corrected in respect of the characteristics dimensions of the structural member; $f(\lambda)$ is the dimensionless function of the corrected crack length termed $K$-calibration.

Using for Eq. (9) the equation from the general error theory [2], and carrying out simple transformations, we obtain

$$\delta_{K^*} = \delta_{\sigma_\infty} + \frac{1}{2} \delta_\lambda + \delta_f.$$  \hspace{1cm} (10)

where $\delta_{K^*}$, $\delta_{\sigma_\infty}$, $\delta_\lambda$, $\delta_f$ are the relative errors of respectively the SIF, nominal stress, actual crack size, and function of $K$-calibration. All these characteristics are determined for the structural member. The value of the relative error in Eq. (10) for the actual crack length $\delta_{\lambda}$ is determined by the accuracy of the method of nondestructive inspection used for