A topical problem is the investigation of the impact strength of multilayered structures in which polymer materials are used. If it is to be solved, it is necessary to know the dynamic properties of these materials in a wide range of strain amplitudes and frequencies and temperatures.

The described method was worked out for the purpose of certifying the oscillation characteristics of plastics [1] but it can also be successfully used for measuring the dynamic properties of other materials including metals. The advantages of the method in comparison with the standard [2] based on [3, 4] are revealed in investigations of very tough materials.

The method is based on the accurate solution of the linear problem of steady-state forced vibrations of a specimen fastened to the vibrator plate and pressed against it from above by some loading weight (Fig. 1). The stress in the layer dy is equal to

$$\sigma = \mu' - E_d \varepsilon',$$

where \(z\) is the displacement, \(z(y, t); \mu\) is the toughness of the material; \(E_d\) is the dynamic modulus of elasticity.

The equation of vibrations of the layer has the form

$$\rho \ddot{z} = \mu z^\prime + E_d z^\prime\prime,$$

where \(\dot{z} = \partial z / \partial t; z'' = \partial^2 z / \partial y^2\).

The solution is sought in the form of a harmonic wave

$$z = [C_1 \exp (\delta_1 y) + C_2 \exp (\delta_2 y)] \exp (i\omega t).$$

The boundary condition for \(y = 0\) is the specified displacement \(z_0\), and on the upper boundary for \(y = h\) the equality

$$\sigma h = M \dot{z} h / F,$$

where \(M\) is the loading weight; \(F\) is the cross-sectional area of the specimen.

The experiments entail the measurement of the amplitudes of the vibration acceleration of the vibration table \(|\ddot{z}_c|\), of the loading weight \(|\ddot{z}_h|\), and the phase shift \(\theta\) of these vibrations when excited with varied frequency \(\omega\). The equations of correlation of the modulus of elasticity \(E_d\) and of the loss coefficient \(\eta = \mu\omega / E_d\) with the mentioned measuring parameters are finally written in the following manner:

$$A \exp (-i\theta) = \frac{2E_d \delta_1 (1 + i\eta)}{[M \omega^2 (1 + i\eta) + E_d \delta_1 (1 + i\eta)] \exp (-i\delta_1 \delta) - [M \omega^2 / F - E_d \delta_1 (1 + i\eta)] \exp (i\delta h)},$$

(1)
TABLE 1. Comparison of the Accurate and the Approximate Solutions

<table>
<thead>
<tr>
<th>n</th>
<th>$\nu$ $\text{Kg/m}^2$</th>
<th>$\nu$ $\text{sec}^{-1}$</th>
<th>$\mu_{d}$ $10^{-6}$</th>
<th>$\eta$</th>
<th>$\delta_{1}$</th>
<th>$\delta_{2}$</th>
<th>$\delta_{3}$</th>
<th>$\delta_{4}$</th>
<th>$\delta_{5}$</th>
<th>$\delta_{6}$</th>
<th>$\delta_{7}$</th>
<th>$\delta_{8}$</th>
<th>$\delta_{9}$</th>
<th>$\delta_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>1380</td>
<td>3.56</td>
<td>1.333</td>
<td>0.300</td>
<td>0.358</td>
<td>1.272</td>
<td>-4.6</td>
<td>0.293</td>
<td>-2.4</td>
<td>0.216</td>
<td>0.203</td>
<td>0.096</td>
<td>0.082</td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
<td>8070</td>
<td>3.52</td>
<td>1.333</td>
<td>0.300</td>
<td>0.256</td>
<td>1.302</td>
<td>-2.3</td>
<td>0.298</td>
<td>-1.2</td>
<td>0.219</td>
<td>0.203</td>
<td>0.096</td>
<td>0.082</td>
</tr>
<tr>
<td>1</td>
<td>0.3</td>
<td>6620</td>
<td>3.505</td>
<td>1.333</td>
<td>0.300</td>
<td>0.210</td>
<td>1.315</td>
<td>-1.4</td>
<td>0.298</td>
<td>-0.8</td>
<td>0.219</td>
<td>0.203</td>
<td>0.096</td>
<td>0.082</td>
</tr>
<tr>
<td>1</td>
<td>0.4</td>
<td>5750</td>
<td>3.50</td>
<td>1.333</td>
<td>0.300</td>
<td>0.182</td>
<td>1.323</td>
<td>-0.8</td>
<td>0.298</td>
<td>-0.6</td>
<td>0.219</td>
<td>0.203</td>
<td>0.096</td>
<td>0.082</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
<td>6250</td>
<td>3.68</td>
<td>1.333</td>
<td>0.300</td>
<td>0.205</td>
<td>1.179</td>
<td>-12.1</td>
<td>0.282</td>
<td>-5.9</td>
<td>0.219</td>
<td>0.203</td>
<td>0.096</td>
<td>0.082</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
<td>4560</td>
<td>3.59</td>
<td>1.333</td>
<td>0.300</td>
<td>0.434</td>
<td>1.248</td>
<td>-5.4</td>
<td>0.290</td>
<td>-3.3</td>
<td>0.219</td>
<td>0.203</td>
<td>0.096</td>
<td>0.082</td>
</tr>
<tr>
<td>3</td>
<td>0.3</td>
<td>3760</td>
<td>3.56</td>
<td>1.333</td>
<td>0.300</td>
<td>0.358</td>
<td>1.272</td>
<td>-4.6</td>
<td>0.293</td>
<td>-2.4</td>
<td>0.216</td>
<td>0.203</td>
<td>0.096</td>
<td>0.082</td>
</tr>
</tbody>
</table>

Before the tests the specimen is held in a thermostated chamber of the vibrator for at least three hours; this time is necessary for endowing the specimen with the specified temperature conditions of the experiment; these conditions are checked continuously during the entire experiment. The vibration measurements by the described method do not take much time. When the arrays $A(\omega)$, $\theta(\omega)$ are directly fed to the computer input, the recording takes several seconds. This guarantees that the specified temperature of the specimen is accurately maintained during the experiment, even with materials with high loss factors. The existing standard (resonance) method [2] requires a considerable amount of time for search and rechecking of the resonance frequency; this inevitably leads to the specimen being heated and to the appearance of an additional error due to the temperature-time dependence of the viscoelastic properties of the material.

The system of two complex equations (1) thus written was solved with the use of the language Alpha-6 (which operates with complex variables) by the network method with fractional step that decreases in successive approximations. The solution on a BESM-6 computer requires $\approx 7$ sec computer time for one variant of measurements.

As first approximation we deal with the case when the length of the strain wave is much greater than the height of the specimen (with $|\delta_{1}|h \ll 1$). Here the following calculation formulas apply:

$$E_{d} = M\omega^{2}h(A \cos \theta - A^2)F/(2A \cos \theta - 1 - A^2).$$

$$\eta = \sin \theta/(A - \cos \theta).$$

With the phase angle $\theta = \pi/2$ these formulas become even more simplified:

$$E_{d} = \frac{M\omega^{2}h}{F} \frac{A^2}{A^2 + 1}; \eta = \frac{1}{A}.$$