LOCAL PULSED LOADING OF COAXIAL CYLINDRICAL SHELLS INTERACTING THROUGH LIQUIDS

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The hydroelasticity problem is usually solved for the cases of the effect of the hydraulic shock on a structure [1]. However, cases of loading structures on the surface which is not in contact with the medium are also of interest for practice [2-4]. In this article, we examine the reaction of two coaxial cylindrical shells, separated with a liquid layer, to pulsed loading applied on part of the surface of the external shell (Fig. 1). The problem is examined in the two-dimensional formulation. Movement of the shells is determined by the equations written in relation to the displacements whereas the behavior of the liquid is described by hydrodynamic equations. The problem is solved by the finite difference method. It is required to examine the effect of the reduction (increase) of the loaded section and also of the nonlinearity of the hydrodynamic equations on the deformed state of the examined hydroelastic system.

1. The liquid is assumed to be ideal. Its motion is described by the following equations of conservation of state (in the polar coordinate system r, θ)

\[ \frac{\partial v_r}{\partial t} = -\frac{1}{\rho} \frac{\partial P}{\partial r}, \quad \frac{\partial v_\theta}{\partial t} = -\frac{1}{\rho r} \frac{\partial P}{\partial \theta}, \]

\[ \frac{\partial P}{\partial t} + \frac{\rho}{r} \left( \frac{\partial (rv_r)}{\partial r} + \frac{\partial (rv_\theta)}{\partial \theta} \right) = 0, \]

where \( v_r, v_\theta \) are the speeds of the particles of the liquid in the radial and annular directions; \( P, \rho \) is the pressure and density of the liquid; \( P_0, \rho_0 \) are the nonperturbed pressure and density of the liquid; \( B \) is a constant; \( \gamma \) is the exponent of the adiabatic curve.

The equation of state is written in Tate's form. This equation can be used to examine the propagation of waves with relatively high amplitudes (to 3000 MPa) in the liquid [1]. The differential equations (1) are written on Lagrange's mesh (the independent variables are represented by the Lagrangian coordinates r, θ whose position in relation to Euler's stationary coordinates is determined during solution of the problem). Thus, the system (1) is nonlinear.
not only because of Tate's equation but also as a result of the fact that the mesh of the Lagrangian coordinates is deformed together with the liquid.

The displacements of the cylindrical shells delineating the liquid we assume to be small and we used the equations of the Kirchoff-Love model

$$\frac{\partial^2 v_a}{\partial t^2} + \frac{\partial w_a}{\partial t} = \frac{(1 - \nu_a^2) \rho_a R_a^2}{E_a} \frac{\partial^2 v_a}{\partial t^2};$$

$$\frac{\partial v_a}{\partial \theta} + \frac{h_a^2}{12 R_a^3} \left( \frac{\partial^4 w_a}{\partial \theta^4} + 2 \frac{\partial^2 w_a}{\partial \theta^2} + w_a \right) + w_a = -\frac{(1 - \nu_a^2) R_a^2}{E_a h_a} \left( q_a + \rho_a h_a \frac{\partial^2 w_a}{\partial t^2} \right).$$

In this case, the shells are numbered in the order of the increase of their radii ($\alpha = 1$ for the internal shell, $\alpha = 2$ for the external shell); $w_1, v_1$ are, respectively, the radial and annular displacements of the shell; $E_a$ is Young's modulus; $\nu_a$ is Poisson's ratio; $\rho_a$ is the density of the material of the shells; $R_a, h_a$ are, respectively, the radius and thickness of the shell; $q_1 = p_1$; $q_2 = q - p_1$.

Part of the external surface of the structure is subjected to the effect of a pressure pulse whose intensity along the angular coordinate varies in accordance with the sinusoidal law (Fig. 1).

$$q(t, \varphi) = g(t) \sin \frac{\pi \varphi}{L} \quad (0 \leq \varphi \leq L),$$

where $L$ is the angle of the loaded section.

The solution of Eqs. (1) and (2) must satisfy the nonoccurrence conditions

$$\frac{\partial w_1}{\partial t} = v_1|_{r = R_1}; \quad \frac{\partial w_2}{\partial t} = v_2|_{r = R_2}. \quad (4)$$

2. The numerical solution of Eqs. (1), (2) is based on the application of Wilkins' method [5] in integrating the hydrodynamics equations and difference relations derived for determination of the displacement of the cylindrical shells [2].

The boundary-value conditions for the external and internal shells (4) are satisfied using various algorithms. Calculations of the external shell in each time step $m$ were carried out in accordance with the following iteration procedure:

a) From the equations of motion of the shell, replacing the value of $p$ in the $m$-th layer ($p_m$) by the pressure in the $(m - 1)$-th layer ($p_{m-1}$), we determine, to a first approximation, the displacements $w_1^{(1)}, v_2^{(1)}$.

b) knowing the displacement of the shell, we determine the speed of adjacent particles of the liquid (4) and then, to a first approximation, the pressure on the shell ($p_m^{(1)}$). The operations given under $a$ are then repeated using the determined value of ($p_m^{(1)}$), and determine, in the second approximation, the displacements $w_1^{(2)}, v_2^{(2)}$. Subsequently, we determine the pressure ($p_m^{(2)}$).

Iterations are continued until the conditions