STRESS INTENSITY FACTORS OF SEMICIRCULAR
AND QUADRANT-SHAPED CRACKS IN POLYNOMIAL LOADING

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Analytical solutions of three-dimensional problems of fracture mechanics have been obtained only for infinite bodies weakened by cracks of conic form [1]. In nonuniform loading, the accurate expressions of the stress intensity factors (SIF) are available for disk-shaped cracks [1], while the solution for analytical cracks is cumbersome [2, 3]. At the same time, considerable interest is attracted in practice by surface and corner defects positioned usually in the zones with steep stress gradients. Problems of this type are usually solved numerically or approximately using several assumptions [4].

Chell [4] used the solution [5] to estimate SIF along the front of a semicircular crack in loading the edges of the crack by the exponential law on the basis of the hypothesis according to which the correction for nonuniformity of the stress field is identical for semicircular and symmetrically loaded circular cracks; Chell's hypothesis [4] was used for calculating stress concentration factors in nonuniform loading of the edges of a semielliptical crack [6]. The identical approach can evidently also be used in calculating SIF for quadrant-shaped defects. However, the accuracy of this hypothesis has not been sufficiently verified by direct numerical calculations.

In this work, we determined the stress intensity factors along the front of semicircular and quadrant cracks situated in a nonuniform stress field, using the weight-function method (WFM) and the finite-element method (FEM). The calculations were carried out for the load

\[ \sigma_{mn}(x, y) = \sigma_0 \left( \frac{x}{a} \right)^m \left( \frac{y}{b} \right)^n \]  

(1)
Fig. 1. Semicircular (quadrant) crack.

Fig. 2. Division of the layer with the semicircular (quadrant) crack into finite elements. (The division contains 440 elements, including 128 square and 312 linear elements, and also 1136 nodes.)

Fig. 3. Distribution of dimensionless stress intensity factors along the front of the quadrant crack in the finite thickness plate: 1) \(a/h = 0.3\); 2) \(a/h = 0.4\).

Fig. 4. Values of the correction for the nonuniformity of the stress field along the front of the semicircular (a) and quadrant-shaped (b) cracks: 1) WFM (approximation \(f_0\) [9]); 2) Chell's hypothesis [4]; 3) data from [10]. (Solid lines were obtained by FEM, \(a/h = 0.3\); broken lines by WFM (the equations for \(f_0\) were taken from the present work)).

for the polynomial degree \(m + n \leq 3\), where \(a\) is the radius of the crack; Oxy is the right-angled coordinate system coinciding with the crack plane, with the Ox axis being directed in the depth of the body (Fig. 1).

According to the weight-function method, SIF in polynomial loading of the edges of the crack \(K_{mn}\) is linked with SIF in uniform loading \(K_0\) by the following expression [7]

\[
\int_\Gamma \frac{K_{mn}K_0}{H} d(\delta S) = \int_\Sigma \sigma_{mn}\delta u_0 dS, \tag{2}
\]

where \(\Gamma\) and \(\Sigma\) are, respectively, the contour and the area of the crack; \(H\) is generalized Young's modulus (\(H = E\) for the planar stress state and \(H = E/(1 - \nu^2)\) for plane strain, \(\nu\) is Poisson's ratio); \(\delta u_0\) is the variance of the crack edge opening displacement taking place when the area of the crack varies by \(\delta S\). The expression for the weight function \(\delta u_0\) for the crack with the elliptical front and the body of infinite dimensions (\(a\) and \(b\) are the half axes of the ellipse) was derived in [7]:

\[
\delta u_0 = \delta u_0|_{\Sigma} + \delta u_0|_{\Gamma}. \]