METHOD OF EXTRAPOLATING INELASTIC DEFORMATIONS FOR THE ACCELERATED CALCULATION OF THE KINETICS OF CYCLIC DEFORMATION

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The calculation of the kinetics of inelastic deformation of structures makes it possible to use the most adequate models of the medium and to obtain the most complete information on the inelastic operation of structures with real, often very complex programs of cyclic thermomechanical action. However, such a calculation is also the most laborious one: before the cyclic deformation of a structure becomes stabilized, the calculation of hundreds of cycles is sometimes necessary [1] (even with up-to-date computer techniques such a calculation may turn out to be feasible). New ways of reducing the computation time are therefore sought, especially by using certain regularities in the inelastic behavior of structures.

For instance, a certain endeavor of the process of deformation under regular cyclic loading toward stabilization has strict directivity: the incompatibility of the change in inelastic deformations in one cycle leads to such a change of residual stresses that in the subsequent cycle the change of inelastic deformation becomes more compatible, and the residual stresses undergo a smaller change. This tendency can be used for predicting the future state of a structure from the results of the calculation of a small number of cycles \( N_b \) by means of extrapolation. The errors can be substantially reduced by confining the extrapolation to a limited number of cycles \( \Delta N \), the calculation of the kinetics during \( N_b \) subsequent cycles corrects the obtained state and yields information for a new extrapolation to \( \Delta N \) cycles, etc.

For calculating gas turbine blades, the authors of [2] suggested extrapolation of the residual stresses; this reduced the time for calculating the kinetics up to the steady state (to between one third and one sixth). However, at the same time there arise certain difficulties with the calculation of cumulated one-sided deformation which may be of great importance in the evaluation of the life of the structure concerned. To eliminate this shortcoming, in the present communication we suggest applying extrapolation of inelastic deformation. Special features of this method are illustrated on the example of calculations of gas turbine disks and blades.

The method of extrapolating residual stresses [2] is based on two conclusions entailed in the use of the model of a cyclically stable medium: for the calculation of the kinetics of inelastic deformation in an arbitrary cycle it suffices to know the initial stresses; in the process of cyclic deformation residual stresses gradually tend to steady-state values. Assuming that the rate of stabilization follows an exponential dependence, it is easy to predict the residual stresses in the subsequent cycles. The parameters of the exponential curve are determined after the calculation of the kinetics in the course of \( N_b \) cycles from the data on the last three cycles; the magnitude of \( \Delta N \) may be of the order of several tens.

A calculation of gas turbine blades (stabilization practically ended at the 250th cycle) showed that with \( N_b = 7 \) cycles and \( \Delta N = 40 \) cycles, no more than 36 cycles (five extrapolations) suffice for determining the double amplitude of inelastic deformation in a stabilized cycle.

The method described earlier in [2] provides considerable savings of time with problems when stabilization proceeds comparatively slowly. Calculations show that these are the cases involving a noticeable cumulation of one-sided deformation. However, to calculate these requires additional calculations with extrapolation and correction. A more economical method is possible: it is based on the extrapolation of the values of inelastic deformation (not stresses). The stresses required for the subsequent calculation of the kinetics are found from the solution of the problem of the elastic deformation of the structure with specified load (corresponding to the instant \( t \) under consideration) and initial deformations found by extrapolation. A distinguishing feature of the extrapolation is that it takes into account the possibility of progressive cumulation of deformations in a stable cycle (addition of a linear term).
Fig. 1. Program for loading the gas turbine blade and disk (T is the temperature at the places indicated by an arrow; n is the rotational frequency; F is the cross-sectional area of the blade; \( \sigma_{p_{\text{max}}} \) is the tensile stress in the calculated section of the blade for \( n = 30,000 \) rpm).

Fig. 2. Inelastic deformations at the end of the cycle in the first (1) and second (2) subelements of the model for points lying in the central part of the blade section; light dots: extrapolation to the 15-th cycle, dark dots: extrapolation to the 20th cycle. (Here and in Figs. 3-5 dashed lines indicate successive calculation of the kinetics.)

Fig. 3. Double amplitude of inelastic deformation in the cycle on the leading edge of the blade, \( \Delta N = 15 \) cycles. (Here and in Figs. 4, 5 solid lines indicate calculation with extrapolations.)

Fig. 4. Kinetics of circumferential residual stresses in the disk (\( \Delta N = 20 \) cycles).

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p(N) = A + B \cdot N + \sum_{k=1}^{k_{\text{max}}} c_k \exp(D_k N).
\]

Here there are only 2 \((k_{\text{max}} + 1)\) parameters which are found after calculation of \( N_{\text{b}} \geq 2 (k_{\text{max}} + 1) \) cycles (in them we pinpoint the values of the inelastic deformations \( p \) at the calculation points corresponding to the chosen instant of the cycle \( t^* \)). The set of exponential curves takes into account the possibility of change of the zones of inelastic deformation with increasing number of cycles.