EXAMINATION OF THE LOADING CONDITIONS
OF SHEARED ELEMENTS

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It is reported in textbooks and manuals that shear deformation (slip) in pure form is virtually impossible to realize: deformation is always accompanied by other deformations, i.e., contact deformation and bending. This fact complicates the examination of shear strains and, consequently, this type of deformation has been studied to a lesser extent than other types. In practice, calculations of shear strains are carried out using an approximate method. For this reason, further investigations into shear strains are not only of theoretical but also of practical interest.

As a first step in a more detailed examination of shear strain, it is useful to determine the loading conditions of sheared elements. The examination of these conditions of loading a single-shear element with a round cross section is described below.

To determine the parameters of loading the sheared element (pin), we shall examine its deformation in the region of the axial gap and in sleeves (Fig. 1). The pin will be classified as a rigid beam of finite length resting on an elastic base (sleeve). In the general case, taking into account the possible formation of axial and radial gaps, we shall have four characteristic loading regions: first 0-1, second 1-2, third 2-3, and fourth 3-4 (Fig. 2).

The above problem will be solved in the following sequence:

1) For each of the above regions we shall derive the differential equation of the bent axis of the pin taking into account the effect of lateral forces on displacement during bending;

2) solving these equations, we obtain for each region the equations of deflections \( y(x) \), angles of rotation \( \theta(x) \), bending moments \( M(x) \), and lateral forces \( Q(x) \); these equations include 16 unknown initial parameters, i.e., \( y_1, \theta_1, M_1, \) and \( Q_1 \) (i = 0, 1, 2, 3), and four additional unknown parameters, i.e., the concentrated moment from the thrust forces \( M \) and the lengths of the typical regions \( b_1, b_2, \) and \( b_3 \);

3) to determine these 20 unknown parameters, we shall examine the conditions of fixing and loading of the pin at its left (section 0) and right (section 4) ends, and also the conditions of smoothness and continuity of the bent axis and the conditions of equality of the internal force factors at the boundaries of the regions (sections 1, 2, and 3).

As indicated by the calculation diagram, in the first and third regions the pin does not rest on the elastic base and, consequently, the load is not distributed. The differential equation for these regions may be written in the following form [1]:

\[
\frac{d^2y(x)}{dx^2} = \frac{1}{EI_x} \left[ M(x) - \beta \frac{d^3y(x)}{dx^3} \right],
\]

where \( \beta = kEJ_x/GF = 0.1806d^2 \) (it is assumed in this case that \( k = (10/9)\mu = 0.3 \)).

Force contact takes place between the pin and the sleeve in the second and fourth regions, i.e., the pin rests on the elastic base and distributed loads \( q(x) \) appear. The differential equation for these regions is derived from Eq. (1) which has been twice differentiated:

\[
\frac{d^4y(x)}{dx^4} = \frac{1}{EI_x} \left[ q(x) - \beta \frac{d^3y(x)}{dx^3} \right].
\]

The theory of bending of beams on an elastic base [1] shows that

\[
q(x) = -k \cdot y(x)^*,
\]

* The theory of a simple elastic base is accepted in this case.
where \( k \) is the stiffness factor of the elastic base; the value of this factor for the given case can be determined using the solution of the contact problem of the elastic compression of the pin and the sleeve [2]:

\[
k = \frac{\partial q}{\partial U} = \frac{E}{2.5915} = 0.3859E.
\]

Consequently, taking into account Eq. (3), Eq. (2) can be rewritten in the form

\[
\frac{d^4y(x)}{dx^4} + \frac{\beta k}{EI_z} \frac{d^2y(x)}{dx^2} + \frac{k}{EI_z} y(x) = 0. \tag{4}
\]

To facilitate subsequent considerations, we introduce the following dimensionless parameters: \( \xi = x/d \) and \( \nu = y/d \), i.e., dimensionless coordinates; \( b_0 = b/d, c_0 = c/d, s_0 = s/d, e_0 = e/d, p = 4P/\pi Ed^3, t = 4Q/4Ed^2, \) and \( m = 4M/\pi Ed^3, \) i.e., the relative values of, respectively, the working length of the pin, the axial gap between the sleeves, misalignment of the sleeves, the radial gap between the pin and the sleeve, shear forces, the lateral force, and the bending moment.

Taking into account the introduced dimensionless parameters, the differential equations (1) and (4) may be written as follows:

\[
\frac{d^4y(\xi)}{d\xi^4} + \frac{\beta k}{EI_z} \frac{d^2y(\xi)}{d\xi^2} + \frac{k}{EI_z} y(\xi) = 0. \tag{5}
\]

\[
\frac{d^4y(\xi)}{d\xi^4} - \frac{\beta k}{EI_z} \frac{d^2y(\xi)}{d\xi^2} + \frac{k}{EI_z} y(\xi) = 0. \tag{6}
\]

The solution of Eqs. (5) and (6) makes it possible to obtain the equations for the deflections \( y(\xi) \), angles of rotation \( \theta(\xi) \), bending moments \( m(\xi) \), and lateral forces \( t(\xi) \) for the examined regions of loading. Omitting the computations associated with the solutions of the differential equations (5) and (6), we shall write their final result.

The first loading region:

\[
v^1(\xi) = -\frac{S_0}{2} + \frac{2\varepsilon_s}{b_0} \xi + 16\rho \left( \frac{\theta_0}{8} - \frac{\theta_s}{6} + 0.1806 \right); \tag{7}
\]

\[
\theta^1(\xi) = \frac{2\varepsilon_s}{b_0} + 16\rho \left( \frac{\theta_0}{8} - \frac{\theta_s}{2} + 0.1806 \right); \tag{8}
\]