INTERACTION BETWEEN A BROKEN-AWAY BLADE AND AN ARMOR RING

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The requirement to ensure the fracture resistance of casings of aircraft gas-turbine engines has resulted in a number of problems which must be solved in order to construct light holding casing members. These problems include the problems of the selection of the material and optimum method of armoring the casing, examination of fracture mechanisms in piercing, special features of the interaction of the broken-away blade, etc.

The present article describes the formulation and method of solving the problem of the interaction of the blade with an armoring member in the case in which the latter can be schematized by a ring and the blade can be schematized by a rod of variable cross section.

The identical problem with certain simplifying assumptions concerned with the interaction between the blade and the ring was examined by Zhukov [1]. Zhukov did not take into account the sliding of the blade in relation to the ring and rotation of the blade in relation to the contact point, the impact was assumed to be inelastic, the ring and the blade were examined as systems with one degree of freedom, etc.

We shall examine the planar movement of the ring and the blade. For this purpose, we shall introduce the stationary coordinate system \( x_0y_0 \), the system \( xoy \) linked with the moving blade, and the moving system \( Tcn \) for the ring (Fig. 1).

The movement of the blade-ring system is determined by the two coordinates \( x_0 \) and \( y_0 \) of the center of gravity of the blade in the \( x_0y_0 \) system, the angle of rotation of the blade \( \psi \) as a solid body, two functions of displacement of the blade \( \xi(x), \eta(x) \) as the deformed rod, and two functions of displacements of the ring \( v(s), w(s) \), where \( s \) is the angular coordinate of the cross section of the ring.

To determine the seven unknown quantities, we shall use three equations of motion of an absolutely hard blade:

\[
M \ddot{X}_b = X_b; \quad M \ddot{Y}_b = Y_b; \quad I \ddot{\psi} = aY_b
\]

(1)

two equations of motion of the blade as the deformed rod:

\[
m_b \dddot{\xi} - T_b = X \delta(x - a);
\]

\[
m_b \dddot{\eta} + M_b \dddot{\xi} = Y \delta(x - a)
\]

(2)

and two equations of motion of the deformed ring

\[
m_1 \dddot{r} - T_1 r^{-1} - M_1 r^{-2} - (M_1 (w^1 + v)) r^{-3} = P_1 \delta(s - q);
\]

\[
m_1 \dddot{w} + M_1 r^{-2} - T_1 r^{-1} - (T_1 (w^1 + v)) r^{-2} = -P_1 \delta(s - q),
\]

(3)

where \( M, I_0 \) are the mass and mass moment of inertia of the blade; \( m_b, m_r \), linear masses of the blade and the ring; \( T_b, M_b, T_r, M_r \), internal force and moment for the blade and the ring, respectively; \( X_0, Y_0, X, Y, P_0, P_r \), projections of the contact force of interaction in the coordinate systems \( x_0y_0, xoy, Tcn \), respectively; \( \delta \), pulsed Dirac's function; \( \varphi \), angular coordinate of the contact point; \( r \), radius of the ring; \( a \), distance from the periphery to the center of gravity of the blade. The nonlinear terms in the left-hand parts of Eqs. (2) and (3) take into account the change in the position of the internal forces in the cross sections of the blade and the ring during their deformation.
We shall now express Eqs. (2) and (3) in relation to the displacements. For this purpose, we shall use the dependence of the internal forces on the tensile $\varepsilon_0$ and bending $\kappa$ strains:

$$T = \Phi_1 \varepsilon_0; \quad M = \Phi_2 \kappa,$$

and also the relationships between the strains and the displacements for the blade

$$\varepsilon_0 = \varepsilon^x + 0.5 (y'^2); \quad \kappa = y^{xx}$$

and the ring

$$\varepsilon_0 = (\varepsilon^g - w) r^{-1} + 0.5 (w') r^{-2} ; \quad \kappa = (w'^2 + v') r^{-2} .$$

The functionals $\Phi_1$ and $\Phi_2$ will be calculated from the equations

$$\Phi_1 = \int \sigma (z) \, dz; \quad \Phi_2 = \int z \sigma (z) \, dz,$$

where $z$ is the coordinate of the fiber of the cross section; $\sigma$ are the normal stresses in the cross section; $F$ is the cross-sectional area.

At low strains

$$\Phi_1 = EF; \quad \Phi_2 = EI,$$

where $E$ is Young's modulus; $I$ is the moment of inertia of the cross section.

If the strains are transferred to the plastic region, to calculate the functionals $\Phi_1$ and $\Phi_2$, it is necessary to have dynamic curves of deformation in the stages of loading, unloading, and repeated loading, and also to know the laws of variation of $\varepsilon_0$ and $\kappa$ during the interaction between the blade and the ring. The values of the functionals $\Phi_1$ and $\Phi_2$ for the deformation curve shown in Fig. 2 for the case of simple loading and for $\varepsilon_0 > 0.5$ (h is the height of the cross section) are given in Table 1.

Taking into account the dependences (4)-(6), the equations of motion (2) and (3) may be written in the following form

$$m \ddot{y} + (\Phi_{1b} (\varepsilon^x + 0.5 (y'^2))) = X \delta (x - a); \quad m \dot{y} + (\Phi_{2b} (x')^2) = Y \delta (x - a);$$

$$m \ddot{v} - r^{-2} (\Phi_{1r} (\varepsilon^g - w + 0.5 r^{-1} (w')^2)) r^{-1} = \Phi_{2r} (w'^2 + v') r^{-3} (\Phi_{2} (w'^2 + v')^2) (w'^2 + v') = P_a \delta (s - q);$$

$$m \ddot{w} + r^{-4} (\Phi_{2r} (w'^2 + v')^2) r^{-2} = \Phi_{1r} (\varepsilon^g - w + 0.5 r^{-1} (w'^2)) r^{-3} (\Phi_{1} (\varepsilon^g - w + 0.5 r^{-1} (w'^2)) (w'^2 + v')^2) = - P_a \delta (s - q).$$

362