The results obtained on the basis of the approximate theory of soft shells differ negligibly and give somewhat understated values.

LITERATURE CITED


FREE DEFORMATION OF A LONG TWO-LAYER NONUNIFORM CYLINDER WITH CONVECTIVE COOLING

V. Ya. Belousov

The present article examines the problem of determining the stresses and radial displacements in a solid two-layer circular cylinder (inside radius $R_1$, outside radius $R_2$, height $H$) in the case when the temperature distribution depends on the radial distance and time $t$. The cylindrical surface is free of stresses. We will assume that there is ideal mechanical contact between the layers of the cylinder. We will ignore changes in the thermophysical and mechanical characteristics of the materials in relation to temperature. The cylinder is cooled from an initial temperature $T_0 = \text{const}$ to the ambient temperature $T_c = \text{const}$. Strains created by cooling of the cylinder will be assumed small. The materials of the layers will be assumed to be uniform, isotropic, and subject to Hooke's law. Also, we will assume that $H/R_2 >> 1$.

The problem consists of determining the functions $\sigma_r^{(i)}$, $\sigma_\varphi^{(i)}$, $\sigma_z^{(i)}$, $\varepsilon_r^{(i)}$, $\varepsilon_\varphi^{(i)}$, $u_r^{(i)}$ in each of the layers of the cylinder. These functions must satisfy the equation of equilibrium

$$r \frac{\partial \sigma_r^{(i)}}{\partial r} + \sigma_r^{(i)} - \sigma_\varphi^{(i)} = 0;$$

the strain compatibility equation

$$\frac{\partial^2 \varepsilon_\varphi^{(i)}}{\partial r^2} = \frac{\partial \varepsilon_r^{(i)}}{\partial r};$$

the relation between the stresses and strains

$$\sigma_r^{(i)} = \frac{1 - \nu^{(i)}}{E^{(i)}} \left( \varepsilon_r^{(i)} - \frac{\nu^{(i)}}{1 - \nu^{(i)}} \sigma_\varphi^{(i)} \right) + \alpha^{(i)} (1 + \nu^{(i)}) T^{(i)};$$

$$\sigma_\varphi^{(i)} = \frac{1 - \nu^{(i)}}{E^{(i)}} \left( \varepsilon_\varphi^{(i)} - \frac{\nu^{(i)}}{1 - \nu^{(i)}} \sigma_r^{(i)} \right) + \alpha^{(i)} (1 + \nu^{(i)}) T^{(i)};$$

the relation between the strains and displacements

$$\varepsilon_r^{(i)} = \frac{\partial u_r^{(i)}}{\partial r};$$

$$\varepsilon_\varphi^{(i)} = \frac{u_\varphi^{(i)}}{r};$$

and the boundary conditions
\[
\frac{d}{dr} \left[ \frac{1}{r^2} \frac{d}{dr} \left( \sigma_r^{(i)} - \frac{\alpha_r^{(i)}}{\rho^2} \frac{\sigma_\rho^{(i)}}{1 - \nu^{(i)}} \right) \right] = 0.
\]

Inserting Eqs. (5), (6) into Eq. (11) convinces us that the constant resulting from integration \(C^{(i)} = 0\). Replacing the strains in this equation by their expressions in terms of stresses (3), (4) and using (1), we find the resolvent equation of the axisymmetric plane problem of thermoelasticity for a two-layer cylinder:

\[
\sigma_r^{(i)} = C_i^{(i)} + \frac{C_2^{(i)}}{\rho^2} - \frac{\alpha^{(i)} E^{(i)}}{1 - \nu^{(i)}} \int_0^\rho \frac{\phi^{(i)}}{\rho^2} \tau^{(i)} \rho \, d\rho,
\]

\[
\sigma_\phi^{(i)} = C_i^{(i)} - \frac{C_2^{(i)}}{\rho^2} - \frac{\alpha^{(i)} E^{(i)}}{1 - \nu^{(i)}} \int_0^\rho \frac{\phi^{(i)}}{\rho^2} \tau^{(i)} \rho \, d\rho,
\]

where \(\alpha = 0, 1\) for the inner and outer layers of the cylinder, respectively.

Introducing the notation \(k = R_1/R_2\) and taking into account (4), (6), (12), and (13), we obtain the following constants of integration from boundary conditions (7)-(10):

\[
C_1^{(i)} = \frac{D_i}{D}; \quad C_2^{(i)} = 0; \quad C_3^{(i)} = \frac{D_2}{D}; \quad C_4^{(i)} = \frac{D_3}{D},
\]

where

\[
D = \frac{(1 + \nu^{(i)})(1 + 2\nu^{(i)})(k^2 - 1) E^{(i)} + (1 + 3\nu^{(i)}) \times}{\frac{\alpha^{(i)} (1 + \nu^{(i)})(k^2 - 2\nu^{(i)} + 1) E^{(i)}}{k^2 E^{(i)}} - \frac{\alpha^{(i)} (1 + \nu^{(i)})(k^2 - 2\nu^{(i)} + 1) E^{(i)}}{k^2 E^{(i)}} - \frac{\alpha^{(i)} (1 + \nu^{(i)})(1 - 2\nu^{(i)}) E^{(i)}}{k^2 E^{(i)}} - \frac{\alpha^{(i)} (1 + \nu^{(i)})(1 - 2\nu^{(i)}) E^{(i)}}{k^2 E^{(i)}}};
\]

\[
D_1 = \frac{\alpha^{(i)} (1 + \nu^{(i)})(k^2 - 2\nu^{(i)} + 1) E^{(i)}}{k^2 E^{(i)}} - \frac{\alpha^{(i)} (1 + \nu^{(i)})(1 - 2\nu^{(i)}) E^{(i)}}{k^2 E^{(i)}} - \frac{\alpha^{(i)} (1 + \nu^{(i)})(1 - 2\nu^{(i)}) E^{(i)}}{k^2 E^{(i)}} - \frac{\alpha^{(i)} (1 + \nu^{(i)})(1 - 2\nu^{(i)}) E^{(i)}}{k^2 E^{(i)}};
\]

\[
D_2 = \frac{\alpha^{(i)} (1 + \nu^{(i)})(1 - 2\nu^{(i)}) E^{(i)}}{k^2 E^{(i)}} - \frac{\alpha^{(i)} (1 + \nu^{(i)})(1 - 2\nu^{(i)}) E^{(i)}}{k^2 E^{(i)}} - \frac{\alpha^{(i)} (1 + \nu^{(i)})(1 - 2\nu^{(i)}) E^{(i)}}{k^2 E^{(i)}} - \frac{\alpha^{(i)} (1 + \nu^{(i)})(1 - 2\nu^{(i)}) E^{(i)}}{k^2 E^{(i)}};
\]

\[
D_3 = \frac{\alpha^{(i)} (1 + \nu^{(i)})(1 - 2\nu^{(i)}) E^{(i)}}{k^2 E^{(i)}} - \frac{\alpha^{(i)} (1 + \nu^{(i)})(1 - 2\nu^{(i)}) E^{(i)}}{k^2 E^{(i)}} - \frac{\alpha^{(i)} (1 + \nu^{(i)})(1 - 2\nu^{(i)}) E^{(i)}}{k^2 E^{(i)}} - \frac{\alpha^{(i)} (1 + \nu^{(i)})(1 - 2\nu^{(i)}) E^{(i)}}{k^2 E^{(i)}}.
\]

Using (4), (12), and (13), from (6) we have the radial displacements in the cylinder:

\[
u_r^{(i)} = \frac{(1 + \nu^{(i)})(1 - 2\nu^{(i)}) \rho R_1}{E^{(i)} \left[ C_1^{(i)} - \frac{C_2^{(i)}}{1 - 2\nu^{(i)}} + \alpha^{(i)} E^{(i)} \left( \frac{1}{1 - 2\nu^{(i)}} \right) \int_0^\rho \frac{\phi^{(i)}}{\rho^2} \tau^{(i)} \rho \, d\rho \right]}.
\]

When \(\rho = 0\), Eqs. (12)-(14) become indeterminate. However, if the temperature at this point