the latter determine the ratio of normal to tangential stresses, the ratio of specific energy consumed in changing
the shape and volume of the body, and naturally, on the state of crack initiation and growth.

LITERATURE CITED

1. G. A. Smirnov-Alyaev, Resistance of Materials to Plastic Deformation [in Russian], Mashgiz, Moscow –

2. G. V. Uzhik and P. F. Koshelev, "Some basic regularities in the variation of static strength in areas of

3. P. F. Koshelev, "Effect of stress state character on stress in areas of stress concentration," Mashino-
vedenie, No. 4, 103-107 (1965).


5. G. A. Smirnov-Alyaev and V. M. Rozenberg, Theory of Plastic Deformation in Metals [in Russian],
Mashgiz, Moscow – Leningrad (1956).

6. V. A. Skudnov and A. G. Kiparisov, "Evaluation of the stiffness of stress state during mechanical tests,"


9. P. F. Koshelev and G. V. Uzhik, "Study of plastic deformation in areas of stress concentration by


REFINED FINITE-ELEMENT SCHEMES FOR
CALCULATIONS ON MASSIVE STRUCTURES

G. G. Zav'yalov, V. V. Kirichevskii,
and A. S. Sakharov

UDC 539.3

Massive structures are widely used in industry, and the best means of computing such structures is the
finite-element one. A major stage in the method is the choice of the coordinate (approximating) functions, which
must provide high accuracy and the best solution for a wide range of problems. This might be attained, on the
one hand, by considering the properties of the body during choice of coordinate functions and on the other by in-
creasing the order of the approximating polynomials. Both of these factors substantially influence the error,
convergence, and stability.

It has previously been shown [1] to be advantageous to discuss the behavior of the body under rigid displace-
ment.

Here we examine the effects of increasing the order of the approximating polynomials representing the
displacements on the error of the calculation; we consider two basic approaches in the finite-element method,
both of which are based on the moment scheme [1].

One scheme consists in using products of Lagrange polynomials of arbitrary degree to approximate the
displacements within a curvilinear three-dimensional finite element, while the other uses spline approximation.
The first employs independent approximation along each of the three directions, e.g., linear, quadratic, and
cubic laws may be used, respectively, for the first, second, and third directions. Other combinations are of
course also possible. The powers used in the coordinate functions are chosen to suit the gradients in the dis-
placements, the geometrical characteristics, the boundary conditions, and other factors.
We consider the derivation of the rigidity matrix for a finite element as a nondegenerate curvilinear parallelepiped; for this purpose we introduce two coordinate systems: the local curvilinear one $\mathbf{x_1x_2x_3}$, to which the finite element is referred along with its mechanical and geometrical characteristics, and the basic Cartesian system $\mathbf{OZ_1Z_2Z_3}$, on which the calculations on the objects are performed [2].

We use the following form for the variation in the strain energy of the element:

$$\delta E = \delta \int \int \int \varepsilon_{ij} \sigma_{ij} dv,$$

where $\sigma_{ij}$ are the components of the stress tensor, $\varepsilon_{ij}$ are the components of the strain tensor, $dv = \sqrt{|g|} dx_1 dx_2 dx_3$, and $g$ is the determinant of the metrical tensor $|g|_{ij}$.

The components of the strain tensor are put in terms of gradients:

**TABLE 1. Results for a Supported Beam**

<table>
<thead>
<tr>
<th>Law</th>
<th>2x2x2</th>
<th>2x2x4</th>
<th>2x2x5</th>
<th>2x2x9</th>
<th>3x3x3</th>
<th>3x3x5</th>
<th>3x3x7</th>
<th>3x3x9</th>
<th>2x2x2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order of equation system</td>
<td>24</td>
<td>48</td>
<td>60</td>
<td>108</td>
<td>81</td>
<td>135</td>
<td>189</td>
<td>243</td>
<td>96</td>
</tr>
<tr>
<td>Run time</td>
<td>14 sec</td>
<td>28 sec</td>
<td>37 sec</td>
<td>55 sec</td>
<td>2 min 20 sec</td>
<td>4 min 31 sec</td>
<td>6 min 14 sec</td>
<td>8 min 18 sec</td>
<td>2 min 48 sec</td>
</tr>
<tr>
<td>Error, % for scheme</td>
<td>148,3</td>
<td>84,7</td>
<td>66,5</td>
<td>43,6</td>
<td>14,8</td>
<td>4,0</td>
<td>0,95</td>
<td>0,1</td>
<td>0</td>
</tr>
</tbody>
</table>