Approximate Calculation of the Elastoplastic Stress-Strain State of a Pipe with a Helical Corrugation

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UDC 539.374

Many studies [1-3] have examined the problem of developing an approximate method to calculate the stress-strain state of self-compensating pipes, which are long pipes with a rectilinear axis and a helical corrugation. We will study the elastoplastic stress-strain state of such pipes when they are subjected to axial tensile forces of different magnitude.

We will examine an initially unstressed, nonuniform, isotropic, self-compensating pipe with a helical corrugation. The nonuniformity of the pipe material may have been caused by its processing. The axis of the pipe coincides with the \( z \) axis of the cylindrical coordinate system \( z, r, \theta \). We will also use the natural orthogonal curvilinear coordinate system \( s, \zeta, \varphi \) connected with the middle surface of the pipe. The \( s \) axis is directed across the corrugation, the \( \zeta \) axis is directed along a normal to the middle surface, and the \( \varphi \) axis is directed across the corrugation. The radius of the pipe will be designated through \( R \), the height of the corrugation through \( \eta \), the distance between the vertices of two adjacent corrugations as \( 2L \), and the angle of inclination of the helix of the corrugation to the pipe axis as \( \delta \).

We will assume that, under the influence of an axial force \( Z \), individual elements of the pipe go beyond the elastic limit of the material. We also assume that pipe elements located an equal distance between two adjacent corrugations are in a momentless stress state characterized by the forces \( N_s, N_\theta, \) and \( N_{s\varphi} \), in the coordinate system \( s, \zeta, \) and \( \varphi \) and by the forces \( N_z \) in the coordinate system \( z, r, \) and \( \varphi \), where \( N_z = -Z/2\pi R \).

The relationship between these forces is given by the equalities [3]

\[
N_s = \overline{N}_s \cos^2 \delta; \quad N_\theta = \overline{N}_\theta \sin^2 \delta; \quad N_{s\varphi} = \overline{N}_{s\varphi} \sin \delta \cos \delta.
\]

We further assume that the stress-strain state of the pipe sections between two adjacent helical surfaces dividing the distance between adjacent corrugations in half is approximately the same as the stress-strain state of a shell of revolution with the radius \( R_0 = R / \cos \delta \) and the length \( 2L \) under the influence of meridional \( N_s \) and shearing \( N_{s\varphi} \) forces and a distributed normal load \( q_\zeta = N_\zeta / R_0 \).

The elastoplastic stress-strain state of the shell modeling the self-compensating pipe will be determined form the Kirchhoff-Love hypotheses in a geometrically linear, quasistatic formulation and from relations of the theory of small elastoplastic strains for strain-hardening materials. The latter relations are linearized by the method of elastic solutions [3]. In this case, the solution of the problem reduces to integration of the following system of eighth-order ordinary differential equations in each approximation [3]:


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Fig. 1. Shell of revolution modeling a pipe with a helical corrugation.

Fig. 2. Distribution of the meridional $\sigma_{S}$ (a) and hoop stresses $\sigma_{\theta}$ (b) and the meridional strains $\varepsilon_{SS}$ (c) along the $S$ axis.

\[
\frac{d\vec{N}}{ds} = A(s) \vec{N} + \vec{f}(s)
\]  

(1)

with the boundary conditions

\[
B_{1}\vec{N}(s_{0}) = \vec{b}_{1}; \quad B_{2}\vec{N}(s_{0}) = \vec{b}_{2}.
\]  

(2)