HYBRID CREEP IDENTIFICATION

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Existing phenomenological theories of plastic strain at high temperatures incorporate the effects of dissipation in macroscopic volumes but do constitute abstract models for the phenomena in microscopic volumes. The dissipation description is based on the assumption that the strain history as a function of macroscopic stress is compatible with physically decaying memory [1]. Other approximations involve assuming internal variables representing the free energy of particles, whose rates of change are expressed by means of evolutionary equations [2, 3].

These phenomena require various abstractions or thermodynamic-continuum concepts for closed systems, which are used in defining the concepts of body, motion, mass and surface forces, stress vector, internal energy, temperature, entropy as a probability measure, heat fluxes, and so on. These abstract concepts cover the main principles of physics, such as the conservation of mass and energy, the conservation of momentum, and the conservation of angular momentum. The response of a material to energy input is described by a constitutinal equation subject to appropriate thermodynamic restrictions, such as those on maximal disorder in the system, which may be expressed via the Clausius-Duhem inequality for the entropy, principles of causality, and the like.

The thermodynamics of closed systems describes dissipation determined by the response of the material to the energy input, which always results in increase in entropy, as well as loss of structure. This is a comparatively recent philosophical approach, i.e., the analysis and synthesis of structures serves in essence merely to define axioms of a dialectic of Socratic type for describing inanimate nature.

In recent years, mathematical methods have been developed for the thermodynamics of open systems, which have been applied particularly to biological and physiological processes, which can respond to constantly changing environments, the general principle being that of homeostasis; this is the capacity to adjust, and to perceive and manipulate data and employ negative feedback. There are also control schemes involving positive feedback, which are equally important, although negative-feedback systems predominate in engineering.

One consequence of this is that entropy may fall in an open system as a result of information processing, which can result in ordering and transition to more highly organized structures, whereas propagation of any damage involves very heavy dissipation, and in particular maximum disorder in a system.

A simple homeostat is described by means of evolutionary equations, which in this case are linear, and which serve to define the control error [4, 5]. These are the so-called internal variables in the thermodynamics of closed systems [6,7], which are related to the evolutionary equations and represent the microstructure of the body. These features describe the system adaptation and the controllable optimization of dissipation. The instantaneous values of such variables are determined by the history of the system, no matter what the values of variables such as the motion (deformation), the temperature, and the other external variables.


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Here we consider a generalization providing a mathematical description of hybrid body identification [8, 9], in which we incorporate the history of the loading and the effects on the constitutional equation of a material. This identification expresses a certain anisotropy in the material in space and time and allows one to extrapolate the observed homogeneous fields. The resulting experimental data on the motion and temperature provide input data for the identification problem. The identification is performed for each particular instant.

This identification has been used in examining damage caused by creep; this may be handled as a non-conjugate physical problem with geometrical nonlinearity [7]. It is assumed that the corresponding functions are smooth, and then the thermodynamic state of the body is described by equivalent equations for the deformation and temperature.

**Mathematical Model for Creep.** We are given a body $B$ bounded by a closed surface $\partial B$ with the initial shape $k_0$. We assume that there is no difference in nature between inertial and gravitational masses.

Complete definition of the positions of all particles gives us the configuration of the body; a particle $X$ in position $X$ at time $t = 0$ moves in time $t$ by deformation to point $x$ (by motion here we mean a continuous sequence of configurations in time):

$$x = x(X, t), \forall X \in B, t \geq 0 \{x\},$$

where $X$ is the coordinate of particle $X$ of the undeformed body in a three-dimensional Euclidean space with respect to the initial configuration $k_0$. The motion described by (1) maps the initial configuration $k_0$ into another configuration $k$.

The neighborhood of particle $X$ in $k_0$ is reflected in a neighborhood of $k$ via

$$dx = F \cdot dX,$$

where $F = \partial x/\partial X$ and the dot denotes the internal product.

A measure of the deformation, which is directly correlated with the intuitive sense of deformation, is the Lagrange deformation tensor $E(x, t)$:

$$E(x, t) = \frac{1}{2}(F \cdot F - I),$$

where $I$ is unit tensor and $T$ denotes a transposed matrix. If the motion is infinitesimal, we have

$$\|x - X\| \ll 1, \forall x \in B, t \geq 0,$$

and the configurations $k_0$ and $k$ differ only by an infinitesimal displacement vector.

The stresses at a point within the body are characterized by the Cauchy stress tensor $T(x)$, which is measured in terms of the current configuration:

$$\tau(x) = T(x) \cdot n(x),$$

where $\tau(x)$ is the vector for the force acting on the surface of the body and having unit normal $n$.

The internal variables [6, 7] are convenient in describing the thermodynamic state of the body under mechanical load over a time $\Delta t$; consequently, we assume that there exists a set $N$ of functions $\{\alpha_i, i = 1, \ldots, N\}$ called the internal variables of the material, which represents the microstructural features (the response function for the phenomenological behavior). If these internal variables satisfy the revolutionary equations, the instantaneous values will determine the history of the other independent variables, and this means that one can incorporate the history of the deformation into the constitutional equation via the response function.

If this is considered as a nonconjugate nonlinear problem, we can take the following as independent state variables: the motion (deformation), the local absolute temperature $\theta > 0$, and the internal variables $\alpha$. The dependent variables are the macrostress tensor $T$, the heat flux $q$, the specific free energy $\psi$, and the specific entropy $\eta$. The thermodynamic state is then defined by a set of state variables:

$$\{u, T, \theta, \alpha\} = U(x, t),$$

where the mass forces $b$ and the internal heat source $r$ are given functions.