STRENGTH CRITERION FOR COMPOSITES BASED ON POLYMER BINDERS AND HOLLOW, SPHERICAL PARTICLES

P. G. Krzhechkovskii

The present work examines the question of a strength criterion for one class of structurally inhomogeneous materials — composites consisting of thin, hollow spherical particles (filler) distributed in a matrix, the materials of which may in turn be a composite. In the use of such materials, stresses may be created which approximates a state of cubic compression; the superposition of shear stresses on a compressive plane-stress state, etc. In evaluating the strength of composites in such stress states on the basis of modern theories of strength for materials differing in their resistance to tension and compression, unsatisfactory agreement is obtained with empirical results in some cases, while in other cases calculations indicate unlimited strength in cubic compression [1-3, etc]. The main shortcoming of these criteria with regard to the case being considered here, noted in [1], is evident: These theories more or less successfully describe stress states in regions lying close to a point of intersection of an interface corresponding to the given theory with a true interface.

In selecting a theory of strength for the materials being investigated, in addition to the requirements formulated in [1, 2], the following should be added: The boundary hypersurface in the stressed space, being convex (the mechanical characteristics of the composite satisfy Drucker's postulate), should be closed in cubic compression, i.e., fracture is possible both in the case of cubic compression and in the case of cubic tension.

We will choose the form of the interface by proceeding on the basis of the character of fracture of the investigated materials. As experiments have shown, the failure of the composite may be related to two parallel and interconnected processes:

a) fracture of inclusions due to loss of either stability or strength as a result of volume deformation;

b) fracture of the matrix, weak in shear.

Based on this, we will assume that the dangerous state for the composite is established with the attainment of energies consisting of part volume-change energy (fracture of the filler does not always lead to fracture of the composite) and part shape-change energy of a certain critical value.

We will write the equation of the interface in the form:

\[ f(P_1, P_2) = c, \] (1)

where \( P_1, P_2 \) are base invariants

\[ P_1 = \sum_{k=1}^{3} a_{kk}; \]

\[ P_2 = \sum_{l=1}^{3} \sum_{k=1}^{3} a_{lk}^2. \]

Let us assume that the relationship between the quantities \( P_1 \) and \( P_2 \), taking into account the character of fracture, takes the form proposed independently by Buzhinskii and Yang [1]:

TABLE 1. Experimental and Theoretical Values of the Mechanical Characteristics of a Composite, Calculated according to Different Theories of Strength

<table>
<thead>
<tr>
<th>Mechanical characteristics, kgf/cm²</th>
<th>Mean test values</th>
<th>accord. to criterion(3a)</th>
<th>accord. to criterion(3b)</th>
<th>accord. to criterion(3c)</th>
<th>accord. to Yang's criterion</th>
<th>accord. to Balandin's criterion</th>
<th>accord. to Schleicher's criterion</th>
<th>accord. to data in [3]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_t$</td>
<td>260</td>
<td>---</td>
<td>---</td>
<td>253 (2.7)</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>690</td>
<td>---</td>
<td>670 (2.9)</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>$\tau_s$</td>
<td>260</td>
<td>266 (2.3)</td>
<td>---</td>
<td>---</td>
<td>245 (5.8)</td>
<td>258 (0.8)</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>$\sigma_h$</td>
<td>1050</td>
<td>---</td>
<td>---</td>
<td>1360 (29.8)</td>
<td>---</td>
<td>1520 (56.0)</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>$\sigma_{2c}$</td>
<td>750</td>
<td>765 (2.0)</td>
<td>730 (2.7)</td>
<td>770 (2.7)</td>
<td>810 (8.0)</td>
<td>1030 (37.3)</td>
<td>1015 (35.3)</td>
<td>566 (29.5)</td>
</tr>
</tbody>
</table>

$\sigma = \sqrt{\tau_s/3}$, being one of the singular points in the Buzhinskii-Yang criterion.

In order to eliminate the above shortcomings, the ultimate hydrostatic strength $\sigma_h$ should be taken as one of the test constants in determining the constants in Eq. (2). The following may be chosen as the two missing test constants:

a) ultimate strength in uniaxial tension $\sigma_t$ and compression $\sigma_c$;

b) ultimate strength in pure shear $\tau_s$ and uniaxial tension $\sigma_t$;

c) ultimate strength in pure shear $\tau_s$ and uniaxial compression $\sigma_c$.

Inserting the ultimate strength of the composite in cubic compression as the required test constant does not introduce additional problems since, first of all, this constant is one of the principal service characteristics of the given materials; secondly, in contrast to ultimate strengths in axial compression, tension, and pure shear, the ultimate hydrostatic strength of composites may be calculated fairly easily theoretically, with an accuracy sufficient for engineering applications, from the geometrical, physical, and mechanical characteristics of the matrix and filler.

The criterion for the limiting state in the expanded form is written as follows:

$$3(p_2 + (a-1)p_1 + b_1) = c.$$  \hspace{1cm} (2)

However, the use of this criterion directly in this form for the materials being investigated may lead to distorted evaluations of strength, since the gradualness of the transition from one type of surface to another is disturbed in the design equation. Thus, for complex EDS-type foams, shear strength lies close to the value $\tau_s = \sqrt{\sigma_t/3}$, being one of the singular points in the Buzhinskii-Yang criterion.

Let us give values of the constants for all three variants: case a:

$$a = 2\frac{3(\eta - \rho) + \eta \rho}{3 - \eta}(3 + \rho); \quad b = \frac{18(\eta - \rho)}{\eta(3 - \eta)(3 + \rho)}; \quad c = \frac{18\eta}{\eta(3 - \eta)(3 + \rho)}.$$  \hspace{1cm} (3a)

case b:

$$a = 2\frac{\sigma_t^2 + 3\sigma_c + \rho}{3 + \rho}; \quad b = \frac{2\sigma_t^2 - 3\sigma_c - \sigma_t}{3 + \rho}; \quad c = 6\sigma_c^2.$$  \hspace{1cm} (3b)

case c:

$$a = 2\frac{\sigma_t^2 - 3\sigma_c + \eta}{3 - \eta}; \quad b = \frac{2\sigma_t^2 - 9\sigma_c^2 + 3}{3 - \eta}; \quad c = 6\sigma_c^2.$$  \hspace{1cm} (3c)