ENERGY DISSIPATION DURING VIBRATION OF MULTILAYER PLATES

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At present the industry uses numerous construction elements, particularly plates, consisting of a large number of strata with various physicomechanical characteristics. The layers of which the plate is made can be arranged in any order and fulfill various functions. Characteristic for such plates is the large influence of the shear deformations in the layers on their work, and also the significant energy dissipation, which manifests itself in the reduction of the resonance amplitudes during vibrations. Therefore, it is obligatory to take into account the above-indicated factors when an analysis of the work of these construction elements is made.

Let us examine the low-frequency vibrations of a plate consisting of layers with various elastic and deformation characteristics (Fig. 1). The layers are combined among themselves in such a way that the following joining conditions are fulfilled:

\[
\begin{align*}
\sigma(z, h_{k-1}) &= \sigma(z, h_{k-1}); \\
\tau_{k}^z(h_{k-1}) &= \tau_{k}^z(h_{k-1}); \\
U_{k}(h_{k-1}) &= U_{k-1}(h_{k-1}); \\
V_{k}(h_{k-1}) &= V_{k-1}(h_{k-1}); \\
\end{align*}
\]

(1)

where \( \sigma \) is the normal stress in the layer; \( \tau_{k}^z \) and \( \tau_{k}^{yz} \) are the tangential stresses in the layer; \( K \), \( U_{k} \), and \( V \) are the displacements of the \( k \)-th layer in the directions of the \( x \) and \( y \) axes respectively; \( h_{k-1} \) are the distances computed from the median plane of the plate to the lower surface of the \( k \)-th layer.

The following kinematic hypotheses are accepted with the purpose of reducing the three-dimensional problem to a plane one:

1) in each layer normal to the median plane the displacement \( W \) does not depend on the coordinate \( z \);

2) for the determination of the deformations, the stresses \( \tau_{x}^{yz} \) and \( \tau_{y}^{yz} \) are not different from the corresponding stresses computed on the basis of the hypothesis that the normal is nondeformable for the total packet as a whole;

3) the stresses \( \sigma_{z} \) are not taken into account in the computation of the stresses \( \sigma_{x} \), \( \sigma_{y} \), and \( \tau_{xy} \).

The hypotheses indicated above are widely accepted in the works of Ambartsumyan [1]. Using hypothesis [2] it is possible to obtain also the equations of the flexure of multilayer shells [3]. (In the following we use some notations from [1] and [3].)

According to hypothesis (1) the tangential stresses are determined from the following expressions of the classical theory of thin plates:

\[
\begin{align*}
\frac{\partial \sigma_{y}^{x}}{\partial x} &= \frac{E_{k}^{x}}{1-\mu_{k}^{y}}, \frac{\partial \phi}{\partial x}; \\
\frac{\partial \sigma_{y}^{z}}{\partial z} &= \frac{E_{k}^{z}}{1-\mu_{k}^{y}}, \frac{\partial \phi}{\partial y},
\end{align*}
\]

(2)

where \( E_{k} \) and \( \mu_{k} \) are the elasticity modulus and Poisson's coefficient of the material of the \( k \)-th layer, respectively and \( \phi(x, y) \) is a certain function of the coordinate surface of the plate under the condition that
Kirchhoff's hypothesis is correct. Integrating within the limits of the layer k and taking into account the joining of the layers we obtain [1]:

$$\tau_{xz} = S_k \frac{\partial \psi}{\partial x}, \quad \tau_{yz} = S_k \frac{\partial \psi}{\partial y},$$

(3)

where

$$S_k = \sum_{i=1}^{h_k} \int_{h_{i-1}}^{h_i} \frac{E_i x}{1-\nu_i^2} dz + \int_{h_{k-1}}^{h_k} \frac{E_k x}{1-\nu_k^2} dz.$$  

(4)

Using the geometrical relationships

$$\frac{\partial U_k}{\partial x} = - \frac{\partial \psi_k}{\partial x} + \gamma_{xz}, \quad \frac{\partial V_k}{\partial x} = - \frac{\partial \psi_k}{\partial y} + \gamma_{yz},$$

(5)

where $\gamma_{xz}$ and $\gamma_{yz}$ are the deformations due to the transversal shear in the layer k, and integrating (5) within the limits of each layer and taking into account (1), (3), and (4) in the form [3], we obtain the following expression for the displacement in the layer

$$U_k = U_0 - \frac{\partial \psi_k}{\partial x} z + \psi_k(z), \quad V_k = V_0 - \frac{\partial \psi_k}{\partial y} z + \psi_k(z).$$

(6)

Here $U_0$ and $V_0$ are the displacements of the points of the coordinate plane of the plate,

$$\psi_k(z) = \sum_{i=1}^{h_k} S_k^i \frac{\partial \psi_k}{\partial i} dz + \sum_{i=1}^{h_{i-1}} S_i^i \frac{\partial \psi_k}{\partial i} dz + \sum_{i=1}^{h_{i-1}} S_i^m \frac{\partial \psi_k}{\partial m} dz - \sum_{i=1}^{h_{m-1}} S_i^m \frac{\partial \psi_k}{\partial m} dz,$$

(7)

where $m$ is the number of the layer in which is located the coordinate plane. The position of the coordinate plane is selected in such a way that the conditions for the stresses $\gamma_{xz}$ and $\gamma_{yz}$ on the upper and lower surfaces of the plate are fulfilled. Thus, in the case when $\gamma_{xz}(h_0) = \gamma_{yz}(h_n) = 0$ the position of the coordinate plane is determined from the condition

$$\sum_{k=1}^{h_k} \int_{h_{k-1}}^{h_k} \frac{E_k x}{1-\nu_k^2} dz = 0.$$  

(8)

The physical conditions relating the stresses to the deformations are determined on the basis of the equations of Pisarenko [2]. For a monoaxial stress state these equations can be written in the form

$$\bar{\bar{\sigma}} = E \left[ \xi \pm \eta \sum_{j=0}^{\chi} C_j^x a^{k-j} \left( \xi y \pm \xi_{m} \delta_j \right) \right].$$

(9)

where $\bar{\bar{\sigma}}$ is the stress for increasing (\(\rightarrow\)) and decreasing (\(\leftarrow\)) deformations; $\xi$ and $\xi_{m}$ are the amplitude values of the deformations; $C_j^x$ are the binomial coefficients; $a$, $\chi$, and $\eta$ are parameters characterizing the amortization characteristics of the material; and $\delta_j = 0$ for odd $j$; $\delta_j = 1$ for even $j$.

In the simplest case, when the hysteresis loop is symmetrical with respect to the coordinate origin, the parameter $a$ is considered to be equal to zero. We note that the parameter $a$ makes more accurate the form of the hysteresis loop, and therefore the following approximations are considered for the solution of the problem by the method of the small parameter in the case when it is introduced. If we limit ourselves to the first approximation for which the final result for the resonance amplitudes does not depend on the shape of the hysteresis loop, then the parameter $a$ can be assumed to be equal to zero or to $\xi_{m}$, which simplifies significantly the subsequent transformations. In this paper we consider only the case $a = 0$ which results in the following relationship:

$$\bar{\bar{\sigma}} = E \left[ \xi \mp \eta \left( \xi_{n} - \xi_{m} \right) \right].$$  

(10)

In order to write the equations for a complex stress state, we assume that the energy dissipation is determined by the components of the stress deviator, while we will take into account the influence of the