CALCULATING ENERGY DISSIPATION FOR THE COMPLEX STRESSED STATE

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Among the questions related to the study of damping of oscillations, the problem of the dissipation of energy in the complex stressed state should be considered as being one of the most fundamental, since in the majority of cases the material in the vibrating components is located precisely in the complex stressed state.

To solve the problem of oscillations in the systems, taking into account the dissipation of energy in the material, use is made of asymptotic methods of nonlinear mechanics developed with reference to a certain type of problem by G. S. Pisarenko [1]. These methods yield solutions both for the simplest cases of oscillations in the system with a single degree of freedom, and for more complex systems. It is necessary to mention that they require the comparison of equations of motion taking into account real (nonlinear) relations between the stresses and deformations. Consequently, in the case of a multidimensional problem it is necessary to use the corresponding equations for the complex stressed state.

N. N. Bogolyubov and Yu. A. Mitropol’ski [2] have given a simple energy interpretation of the formulas in the asymptotic method by means of which it is possible to obtain the first and subsequent approximations without compiling accurate equations for the motion of the system. Yu. A. Mitropol’ski [3] showed the possibility of using this method for solving problems connected with oscillations in systems with distributive parameters in the case of single frequency oscillations. According to this method the equations determining the amplitude \( a \) and the phase \( \theta \) of the oscillations in the solution of the first approximation have the form

\[
\frac{da}{dt} = \frac{1}{\omega M_0} \cdot \delta A; \quad \frac{d\theta}{dt} = \omega + \frac{1}{\omega M_0} \cdot \frac{\delta A}{\delta a},
\]

where \( \omega \) is the inherent frequency of oscillations corresponding to the examined form; \( M \) is the general mass of the system representing the double form of the kinetic energy of the system, in which the total rate of change in the forms of oscillations corresponds to the frequency examined; \( \delta A \) is the average virtual work for the cycle which would be done by the total exciting forces (the forces of dissipation and the external forces) in the schedule of sinusoidal oscillations on the virtual increases in the common coordinates, corresponding to a change in the amplitude and phase of oscillation [3].

A thought arises about the use of this method for calculating oscillations in a system, the material of which is located in the complex stressed state, and the dissipation of the energy is determined by the entire combination of stresses at each point of the volume.

Assuming that, with a sufficiently large number of stresses, the dissipation of the energy is determined in the main by microplastic deformations, the development of which is connected with tangential stresses, we shall take the intensity of the tangential stresses as the general forces, and the overall displacements we shall assume to be the intensity of shear deformation.*

The relationship between the intensities we shall represent in the form of an equation of the hysteresis type

* According to V. V. Novozhilov [4] the intensity of the stress is connected with the mean square value of the tangential stresses developed at a given point.


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\[ \tau_0 = G\gamma_0 + GRf\left(\frac{\gamma_0}{\gamma_{0m}}\right)\gamma_{0m}, \]

where \( \tau_0 \) is the intensity of the tangential stresses, \( \gamma_0 \) and \( \gamma_{0m} \) are the current and amplitude values of the intensity of shear deformation, \( G \) is the shear modulus, \( f(\gamma_0/\gamma_{0m}) \) is a function determining the shape of the hysteresis loop in the coordinates \( \tau_0 - \gamma_0 \), and \( \beta_0 \) and \( n \) are damping parameters.

The form of the loop and the parameters of the damping are determined from tests in conditions of pure cyclic shear.

With one and the same stress intensity the states may be distinguished on account of the average stresses \( \sigma_0 \), which apparently affect the energy dissipation in the material. This is indicated in particular by the difference between the decrements of the oscillations with pure shear and tension-compression [5-7]. If we use the concept of logarithmic decrement of oscillations, it is possible to write the following equality:

\[ \delta = \delta_0 + \delta', \]

where \( \delta \) is the "full" decrement of the oscillations in the system, \( \delta_0 \) is the tentative decrement which would occur in the case of the absence of the influence of the average stress and other factors, \( \delta' \) is the "additive", taking into account the influence of various factors (background of dissipation, average stress, etc.).

Representing the oscillation decrements as the area of the hysteresis loop,

\[ \delta = \frac{\Delta W}{2W}; \delta_0 = \frac{\Delta W_0}{2W}, \]

where \( \Delta W \) is the energy scattered in the system, \( \Delta W_0 = \int\tau_0d\gamma_0, W \) is the potential energy of deformation, and using equality (3) we obtain

\[ \Delta W = \Delta W_0 \lambda. \]

Here \( \lambda \) is the parameter reflecting the change in the area of the hysteresis loop in the coordinates \( \tau_0 - \gamma_0 \) in accordance with the functional relationship, entering into \( \delta' \) (Fig. 1), \( \lambda = 1 + (2W/\Delta W_0)\delta' \). In the special case this relationship may take the form

\[ \delta' = \delta_1 + k\sigma_0^p, \]

where \( \sigma_0 \) is the average normal stress; \( \delta_1, k, \) and \( n \) are constants which can be determined from a comparison of the relationships between the oscillation-amplitude stress decrements in conditions of pure shear and tension-compression.

To locate the complete excitation work we record the elementary virtual work of the forces \( \tau_0 dV \) at the virtual displacement \( \delta \gamma_0 \), and we then integrate with respect to the volume of elastic elements in the system

\[ \delta A = \int \tau_0 \delta \gamma_0 dV. \]

Then we represent the intensity of the deformations in the form

\[ \gamma_0 = \bar{\gamma}_0 a \cos \theta, \]

where \( \gamma_0 a = \gamma_{0m} a; a \) is the amplitude of the oscillations at a certain point in the system. Then the increase in the intensity of the shear deformation is

\[ \delta \gamma_0 = \bar{\gamma}_0 \cos \theta \delta \alpha - \bar{\gamma}_0 a \sin \theta \delta \theta. \]

The average virtual work per cycle of oscillation is

\[ \delta \bar{A} = \frac{1}{2\pi} \int_0^{2\pi} \tau_0 \bar{\gamma}_0 \cos \theta \delta \alpha - \bar{\gamma}_0 a \sin \theta \delta \theta dV, \]

Having determined from Eq. (10) the values of \( \delta \bar{A}/\delta \theta \) and \( \delta \bar{A}/\delta a \), and putting them into Eq. (1) we have

\[ \frac{da}{dt} = \frac{1}{2\pi M_0} \int_0^{2\pi} \tau_0 \bar{\gamma}_0 \sin \theta d\theta dV; \quad \frac{d\theta}{dt} = \omega = \frac{1}{2\pi M_0} \int_0^{2\pi} \tau_0 \bar{\gamma}_0 \cos \theta d\theta dV. \]