Many research workers have studied the creep of bent bars or beams. Analysis of existing papers suggests that the real laws governing the long-term ductility of a bent bar are reflected most closely by the solutions obtained on the basis of hardening theory [1-3]. However, these solutions make no allowance for the stress redistribution arising from the susceptibility of the material to damage during the creep process; hence they fail to predict the development of accelerated creep.

In this paper we shall consider the creep associated with the pure bending of a straight beam on the basis of hardening theory, in the form proposed by Yu. N. Rabotnov et al. [4] and by one of the authors [5]:

\[
\dot{\varepsilon} = m \varepsilon^{1-n} \exp(\alpha \sigma_t),
\]

where \( \sigma_t \) is the true stress in the elementary area considered (allowing for a reduction in the effective area owing to the formation of pores and cracks during the creep process),

\[
\sigma_t = \sigma_e (1 + k \varepsilon);
\]

\( \sigma_e \) is the effective stress, \( \varepsilon \) and \( \dot{\varepsilon} \) are the relative plastic deformation (strain) and the relative rate of deformation at time \( t \); \( \alpha, m, n, k \) are the heat-resistance characteristics of the material at the specified temperature.

Fig. 1. Bending creep curves allowing for damage to the material (continuous curves) and without allowing for this (broken curves): 1) \( \sigma_{\text{max}} (0) = 50 \, \text{kg/mm}^2 \); 2) \( \sigma_{\text{max}} (0) = 35 \, \text{kg/mm}^2 \).

Fig. 2. Dimensionless true and effective stress distributions (continuous and broken lines respectively) in the cross section of the beam at different moments of time: 0) Initial stress distribution; 1) 2 h; 2) 24 h; 3) 120 h; 4) 340 h; 5) 416 h.
Equation (1) has been subjected to extensive experimental verification [5–8], as a result of which it has been shown to agree closely with experimental data. In particular, for \( \sigma_0 = \text{const} \) Eq. (1) describes a creep curve with three sections.

It has been established experimentally [9–12] that in the first section the creep curves respectively associated with extension and compression practically coincide. Hence the heat-resistance characteristics \( \alpha, \beta, \gamma, \) and \( \delta \) are independent of the direction of deformation [13], while the value of \( k \), the characteristic of the susceptibility of the material to damage, is greater for extension, i.e., \( k_{\text{ext}} > k_{\text{com}} \). However, for many heat-resistance alloys we may approximately consider that \( k_{\text{ext}} = k_{\text{com}} \), as a result of this assumption a certain error appears in the deformation (strain) values, but the solution of the problem is greatly simplified.

For simplicity we may consider that the material resists creep equally in extension and compression \( (k_{\text{ext}} = k_{\text{com}} = k) \), the cross section of the beam has at least one symmetry axis, and the beam bends in the principal plane perpendicular to the symmetry axis.

In this case the equilibrium condition may be expressed in the form

\[
2 \int_0^h \sigma_y b dy = M, \tag{3}
\]

where \( 2h \) is the height of the cross section, \( b \) represents the dependence of the width of the section on the coordinate \( y \), \( b = b(y) \), and \( M \) is the bending moment.

The geometrical condition will be

\[
\frac{\sigma_y}{E} + \varepsilon = \chi y, \tag{4}
\]

where \( E \) is the elastic modulus of the material and \( \chi \) is the curvature of the beam axis.

In the general case the initial stress-strained state may be elastic-plastic. Then the distribution of the plastic deformation over the cross section should be specified for \( t = 0 \), i.e., \( (0) = \phi(y) \), together with the damage factor \( k_0 \) of the material under the conditions of preliminary deformation. Equation (2) then takes the form

\[
\sigma_t = \sigma_0 [1 + k_0 (0) + k_0 (t)].
\]

On allowing for the initial deformation no fundamental changes are introduced into the solution of the problem; hence without complicating the solution we may consider the initial deformed state as elastic.

The system of equations (1) to (4) reduces to a single integrodifferential equation in \( \varepsilon \):

\[
\dot{\varepsilon} = m \varepsilon^{1-n} \exp \left[ aE \left( k_0 \varepsilon - \varepsilon \right) \right], \tag{5}
\]

where

\[
\chi = \frac{M}{E} + 2 \int_0^h \frac{by}{1 + k_0} dy.
\]

Since Eq. (5) can only be solved numerically, we replace the integrals in expression (6) by finite sums, using one of the well-known quadrature formulas. For \( l + 1 \) nodes in the integration section, expression (6) reduces to the function \( \chi = f(M, \varepsilon_0, \varepsilon_1, \ldots, \varepsilon_l) \).

For equidistant nodes and \( \varepsilon_0 = 0 \) (deformation of the neutral layer) we arrive at a system of \( l \) ordinary differential equations of the first order:

\[
\dot{\varepsilon}_i = me^{1-n} \exp \left[ aE \left( \frac{i}{l} h \chi - \varepsilon_i \right) \right], \tag{7}
\]

where \( i \) varies from 1 to \( l \). The system (7) may be solved by one of the well-known difference methods or by a technique of the Runge–Kutta type.