The deformation processes taking place in materials subjected to thermal cycling and the conditions of their ultimate failure depend on a number of factors as indicated in many earlier papers [1-3, 5-8].

In this paper we shall consider the effect of the thermal parameters and the boundary conditions on the mechanics of deformation and hence on the rupture conditions of materials on subjecting to repeated thermal loading. In order to establish the corresponding laws of deformation we must know the relationship between the stresses and strains at any given stage of repeated thermal loading. The problem of determining the physical relationships describing the actual behavior of a material subjected to alternating ductile flow as a result of thermal cycling is extremely complex, in view of the nonlinear character of the relationship between the stresses and strains acting, on the one hand, and the various phenomena arising as a result of the conversion of the material into the ductile state on the other.

In order to determine the relation between stresses and strains in each cycle of loading, stress/strain diagrams were constructed by a method specially developed for the purpose [4-5]. Some of these are shown after the first thermal cycle in Fig. 1a.

We see from Fig. 1a-c that for all the materials studied the stress/strain diagrams (hysteresis loops) differ both in size and in shape; in addition to this, all the stress/strain diagrams change their shape and size from cycle to cycle and remain open until a certain number of cycles have been passed (Fig. 1b). Our experimental results showed that the stress/strain diagrams had no linear parts (loading and unloading), and under conditions of "thermally-stable" equilibrium were of a parabolic nature. The falling and rising branches of the hysteresis loop at any N-th cycle of thermal loading may be described by the equations

$$\sigma_N (\varepsilon) = E_N \left( \varepsilon - \frac{\Delta \varepsilon}{2} \right) + \frac{E_N g_N}{P_N} \left[ (\Delta \varepsilon - \varepsilon) + \frac{\Delta \varepsilon^N}{2} \right] + \frac{1 + r_N}{1 - r_N} E_N \frac{\Delta \varepsilon}{2} \left( 1 - \frac{E_N}{P_N} \Delta \varepsilon^{N-1} \right);$$  

$$\sigma_N (\varepsilon) = E_N \left( \varepsilon - \frac{\Delta \varepsilon}{2} \right) - \frac{E_N g_N}{P_N} \left( \varepsilon - \frac{\Delta \varepsilon^N}{2} \right) + \frac{1 - r_N}{1 + r_N} E_N \frac{\Delta \varepsilon}{2} \left( 1 - \frac{E_N}{P_N} \Delta \varepsilon^{N-1} \right) + E_N \left( \varepsilon - \frac{\Delta \varepsilon}{2} \right) (\delta_{pl})_N,$$

where $p_N$, $g_N$ are the ductility parameters of the material; $\varepsilon$, $\varepsilon$ are the current values of the deformation (strain) in the odd and even half cycles; $\Delta \varepsilon$ is the range of elastic-plastic deformation per half cycle, which remained constant over the whole extent of the sample testing under the conditions employed; $r_N$, $r_N$ are the cycle characteristics in the odd and even half cycles; and $(\delta_{pl})_N = (\delta_{pl})_N + (\delta_{pl})_N$ is the residual plastic deformation for the N-th cycle.

The value of $(\delta_{pl})_N$ may be either greater than, less than, or equal to zero, depending on whether the material in question undergoes fatigue hardening or softening, or alternatively acts in the "ideal" manner, under the conditions envisaged. Analysis of the deformation mechanics of the materials tested showed that, after a certain number of thermal cycles ($N = N^*$), an "external" stationary state set in under which $(\delta_{pl})_N = N^*$ = 0, so that in subsequent cycles of alternate thermal loading the stress/strain diagrams repeated themselves. In this case the diagram constituted a closed hysteresis loop (Fig. 1c), the falling and rising branches of which were given by the equations...
Fig. 1. Stress/strain diagrams after the first thermal cycle 100°C \(\Rightarrow\) 700°C under conditions of severe loading (a), after several thermal cycles, for material subject to fatigue hardening (b), and after reaching a steady state for the thermal cycle 100°C \(\Rightarrow\) 700°C (c). (The numbers indicate the characteristic points of the stress/strain curve.)  

In the present case the parameters \(p_N \geq N^*\), \(\varepsilon_N \geq N^*\) change very little from cycle to cycle. For certain materials and thermal conditions, the ductility parameters of the material in the steady state are presented in Table 1.  

In the "stabilized" state there is an accumulation of damage as a result of the alternating plastic flow (yield) of the material. However, it is important to remember that, in addition to the microscopic stabilization of the stresses and strains, there is an increase in the nonuniformity of the strain distribution over the microscopic regions, and this impedes our judgment as to the stabilization of the stressed state. Analysis of the stress/strain diagrams of metals subjected to repeated thermal loading enabled us to establish the laws governing the changes in stress, plastic deformation, and amount of energy dissipated from cycle to cycle. The range of stress fluctuation in each odd and even half cycle was determined from the following formulas for the \(N\)-th cycle:

\[
\Delta \sigma_{N}^{(n)} = E_N \Delta \varepsilon \left(1 - \frac{\varepsilon_N}{p_N} \Delta \varepsilon^{p_{N-1}}\right);
\]

\[
\Delta \sigma_{N}^{(m)} = E_N \Delta \varepsilon \left(1 - \frac{\varepsilon_N}{p_N} \Delta \varepsilon^{p_{N-1}}\right) \pm E_N (\delta_{pl})_{N}.
\]

For materials undergoing fatigue hardening, the range of stress increases monotonically from half cycle to half cycle in every complete cycle (Fig. 2), and the range of stress in one half cycle (even) is greater than that in the other (odd). However, for a certain \(N = N^*\), the range of stress variation in the even half cycle differs negligibly from that in the odd half cycle, and naturally all the curves merge into one (Fig. 2). Here

\[
(\delta_{pl})_{N} \geq N^* = 0,
\]

and

\[
\Delta \sigma_{N}^{(m)} = \Delta \sigma_{N}^{(n)} = \text{const} = \Delta \sigma^*.
\]

The change in the cyclical stress \(\Delta \sigma\) from half cycle to half cycle for \(\Delta \varepsilon = \text{const}\) produces a change in the plastic deformation from cycle to cycle. In each temperature half cycle the amount of plastic deformation depends on the ductility parameters of the material; experiments shows that it has the form

\[
(\delta_{pl})_{N} = \frac{\varepsilon_N}{p_N} \Delta \varepsilon^{p_{N}} - (\delta_{pl})_{N}.
\]

For a certain number of thermal loading cycles \((N = N^*) (\delta_{pl})_{N^*} = 0\), and the absolute value of the plastic deformation in the even half cycle equals that in the odd half cycle:

\[
|\delta_{pl}^{(m)}|_{N = N^*} = |\delta_{pl}^{(m)}|_{N = N^*} = |\varepsilon_{pl}|.
\]

The results of our investigations into the kinetics of the stresses and strains (plastic deformations) in various materials over a wide range of variation of the thermal-cycling parameters