We consider massive solids of revolution in a system of axisymmetric orthogonal coordinates $\alpha, \beta, \theta$ [1].

In the meridional plane, along with the rectangular coordinates $z$ and $\rho$, there is a grid of mutually orthogonal curvilinear coordinates $\alpha, \beta$. The meridional angle $\theta$ is reckoned in the $x, y$ plane normal to $z$, from the $x$ axis to the $y$ axis (Fig. 1). The Lamé coefficients $H_1$, $H_2$, and $H_3$ do not depend on $\theta$. Along the rotation axis $z$, we have $H_3 = 0$.

The two coordinate systems are related by

\[ \beta = \rho \cos \alpha \]

\[ \phi = 0 \]

\[ r = \rho \sin \alpha \]

\[ \varphi = \theta \]

Fig. 1. Solid of revolution in curvilinear triorthogonal coordinates.

Fig. 2. Schematic representation of the effect of concentrated surface stresses on a solid of revolution.
Using this dependence, we can conformally map the \( z, \rho \) plane onto the \( \alpha, \beta \) plane in such a manner that the mutually orthogonal curves \( \alpha = \text{const} \) and \( \beta = \text{const} \) on the \( \alpha, \beta \) plane form a rectangular Cartesian system in which the problem is conveniently solved (Fig. 1).

The shear modulus \( G = E/(1+\nu) \), the Poisson ratio \( \nu \), and the coefficient \( \alpha T \) of linear thermal expansion of the material are functions of the coordinates \( \alpha, \beta \).

We assume the solid is acted upon by distributed surface stresses which vary in some manner in the meridional and azimuthal directions and which are functions of the coordinates. At each point on the solid surface, these stresses may be represented by three components: \( q_\alpha (\beta, \theta) \) normal to the surface, \( q_\beta (\beta, \theta) \) tangent to the meridian, and \( q_\theta (\beta, \theta) \) parallel to the meridian (Fig. 1); in turn, these components can be approximated in the azimuthal direction by trigonometric series:

\[
q_\alpha (\beta, \theta) = \bar{q}_\alpha (\beta) \left( \sum_{k=0}^{n} a_k \cos k\theta + \sum_{k=0}^{n} a_k' \sin k\theta \right);
\]

\[
q_\beta (\beta, \theta) = \bar{q}_\beta (\beta) \left( \sum_{k=0}^{n} b_k \cos k\theta + \sum_{k=0}^{n} b_k' \sin k\theta \right);
\]

\[
q_\theta (\beta, \theta) = \bar{q}_\theta (\beta) \left( \sum_{k=0}^{n} c_k \cos k\theta + \sum_{k=0}^{n} c_k' \sin k\theta \right).
\]

If the stress is written in terms of symmetric and antisymmetric components with respect to the meridional plane adopted as the origin for \( \theta \), then terms in the trigonometric series (2) containing only cosines or only sines will be retained.

An arbitrary temperature field can also be represented in a Fourier series:

\[
T (\alpha, \beta, \theta) = \bar{T} (\alpha, \beta) \sum_{k=0}^{n} \cos k\theta
\]

(3)

(4)

When a solid of revolution is acted upon by concentrated or local stresses, it is convenient to expand these stresses in trigonometric polynomials. We consider the effect of concentrated surface stresses in the meridional plane adopted as the origin for \( \theta \). We consider \( m \) equidistant meridional planes separated by \( \theta = \alpha_0 = 2\pi/m \). The stress \( P_1 (\beta, \theta = 0) \) is expressed in terms of a trigonometric polynomial [3]:

\[
P_1 (\beta, \theta = 0) = \bar{P}_1 (\beta) \sum_{k=0}^{\text{even } m} d_k \cos k\alpha_0
\]

where \( k \) is the polynomial index; \( n \) is the position number; and \( \alpha_0 \) is the cyclic angle.

There is the following number of terms in this cosine polynomial:

\[
j = \frac{m}{2} + 1 \text{ for even } m;
\]

\[
j = \frac{m+1}{2} \text{ for odd } m.
\]

The coefficients \( d_k \) in sum (4) are calculated extremely simply:

\[
d_k = \frac{1}{m} \text{ for } k = 0, k = \frac{m}{2};
\]

\[
d_k = \frac{1}{m} \text{ for } k = 1, \ldots, \frac{m-1}{2};
\]

\[
d_k = 0 \text{ for other } k.
\]