EFFECTIVE USE OF COMPOSITE MATERIALS DIRECTIONALLY REINFORCED WITH HOLLOW FIBERS

G. A. Van Fo Fy and V. V. Klyavlin

We shall determine the effectiveness of using hollow fibers as reinforcing elements of composite materials on the basis of a doubly periodic model of directionally reinforced materials [1]. The connection between the stresses, displacements and complex potentials are established with relationships of the plane elasticity theory [2]

\[
\begin{align*}
\sigma_{33} + \sigma_{22} &= 2(\Phi(z) + \overline{\Phi}(z)); \\
\sigma_{33} - \sigma_{22} + 2i\sigma_{23} &= 2(\overline{Z}'(z) + \Psi(z)); \\
\sigma_{11} &= 2v(\Phi(z) + \overline{\Phi}(z)); \\
20(u_2 + iu_3) &= \kappa \Phi(z) - z \Phi(z) - \psi(z).
\end{align*}
\]

Continuity of the displacements at the fiber–matrix interface, equality of the normal and shear stresses in the fiber and the matrix on the fiber boundary, and freedom from stresses on the inner boundary of the hollow fiber are taken as the boundary conditions. In addition, in the case of longitudinal shear we must ensure that the average stresses on the boundary of an elemental cell of the material are equal to the integral of the stresses along the boundary of this cell [3]. For transverse shear, and longitudinal and transverse tension we must ensure that the principal vector and the principal moment of all forces on the boundary of an elemental parallelogram are zero.

Representing complex potentials for the matrix of the medium in series of Weierstrass functions, we can satisfy the symmetry and double periodicity conditions of the stress state. The stress–strain state in the reinforcement is found in terms of Laurent series. The method of solution is considered in sufficient detail in [3, 4]. In this work we shall establish the relationship between the stresses in the microstructure of directionally reinforced materials and the inside radius of the hollow fiber for various degrees of reinforcement.

We denote the dimensionless inside radius of the fiber by \( q = \frac{\varepsilon}{\lambda} \), where \( \varepsilon \) and \( \lambda \) are respectively the inside and outside radii of the hollow fiber.

On the basis of an analysis of microstress field calculations, it has been established that in materials reinforced with hollow fibers, with \( q < 0.5 \), the microstress concentration factor, for longitudinal and transverse shear and transverse tension, has a maximum value at the fiber–matrix interface. Its value for longitudinal and transverse shear and transverse tension increases as \( q \) increases [4, 5]. However, the weight of a unit volume of the composite decreases at the same time.

To assess the influence of variation of the specific weight of a composite reinforced with hollow fibers, as \( q \) and the stress concentration in the microstructure increase, we introduce the effectiveness of use factor of the material

\[
\gamma_{pik} = \frac{k_{pik}}{k_{pl00} G_{9}} (p = N, T; i, k = 1, 2, 3),
\]

where $k_{pik}$ is the maximum concentration factor of microstructural stresses in the composite reinforced by solid fibers; $k_{pik0}$ is the same factor for the hollow fibers, while $G$ and $G_0$ are the weights of a cell of the composite with solid and hollow fibers, respectively. The quantities $k_{pik}$ and $k_{pik0}$ equal the maximum value of the stresses normal ($p = N$) and tangential ($p = T$) to the boundary of the fiber, when unit average stresses act, and they are determined from the relationships (1).

Since for longitudinal shear the fluctuating stresses caused by the difference in Poisson's ratios of the filler ($\nu_\alpha$) and the matrix ($\nu_\beta$) are considerably below the stresses obtained from the mixture law [6], the value of the effectiveness of use factor is given by the simple relationship

$$\gamma_{N} = \frac{\frac{A}{k_{pik} + \eta} E_a + \eta}{\frac{A}{k_{pik0} + \eta} E_s + \eta}.$$ (3)

Here $A = (\frac{\nu_\alpha + \nu_\beta}{1 - \nu_\alpha \nu_\beta})[\frac{1}{k_{pik} + \eta} E_a + \frac{1}{k_{pik0} + \eta} E_s]^{-1}$; ($\gamma_a$ and $\gamma_s$ are the specific weights of the material of the fiber and matrix); $k_{pik} = \frac{\nu_\beta}{\nu_\alpha} \frac{x}{\cos \alpha}$ ($\omega$ is the period of the grid; $\alpha$ is the grid angle [4]); $\eta = 1 - \xi$ is the volume content of the filler and the matrix; $E_a$ and $E_s$ are Young's moduli of the reinforcement and the matrix.

We see from the relationship given above that for $\gamma_a \gamma_s^{-1} = E_a E_s^{-1}$ the value of $\gamma_{N}$ does not depend on $\eta$ for all $\xi$. If $\gamma_a \gamma_s^{-1} < E_a E_s^{-1}$ (which is observed in glass and metal reinforced plastics), then $\gamma_{N}$ decreases as $\eta$ increases. The rate of reduction increases with the difference $E_a E_s^{-1} - \gamma_a \gamma_s^{-1}$.

The variation of the effectiveness of use factor of the material for longitudinal shear $\gamma_{T13} = \gamma_{T12}$ is presented in Fig. 1. As is seen from the figure, for longitudinal shear the effectiveness of use factor decreases as $\eta$ increases. At the same time effectiveness is higher for highly filled composites than for poorly filled composites. In metal reinforced plastics ($E_a = 0.22 \cdot 10^5$ kgf/cm$^2$, $E_s = 0.31 \cdot 10^5$ kgf/cm$^2$, $\nu_\alpha = 0.23$, $\nu_s = 0.382$, $\gamma_a = 7.8$ g/cm$^3$, $\gamma_s = 1.2$ g/cm$^3$) effectiveness is higher than in glass reinforced plastics ($E_a = 0.7 \cdot 10^8$ kgf/cm$^2$, $E_s = 0.31 \cdot 10^5$ kgf/cm$^2$, $\nu_\alpha = 0.2$, $\nu_s = 0.382$, $\gamma_a = 2.5$ g/cm$^3$, $\gamma_s = 1.2$ g/cm$^3$).

The type of the structure exerts a considerable effect on the factor $\gamma_{T13} = \gamma_{T12}$ (see curves b and b', c and c'). Effectiveness of use of material for transverse shear and transverse tension of a composite must be assessed both for the normal stresses $N$ and the shear stresses $T$, since $N \approx T$. On the basis of an analysis of calculated results, it has been established that $\gamma_{T23}$ decreases as $\eta$ increases. The maximum value of $\gamma_{T23}$ is reached in composites with $\xi = 0.4$. The effectiveness of use factor of the material with respect to normal stresses, for transverse shear, increases continuously as $\eta$ increases when $\xi < 0.45$. In highly filled composites ($\xi > 0.6$) $\gamma_{N23}$ reaches a maximum for $\eta \approx 0.7$; after this it drops sharply. This is observed both in glass and metal reinforced plastics.