INVESTIGATION OF STRESSED STATE OF A
VISCOELASTIC HOLLOW THICK-WALLED CYLINDER
UNDER CONDITIONS OF NONLINEAR CREEP

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Let us consider a long hollow cylinder of a viscoelastic material, enclosed in an elastic shell, subjected to an internal pressure \( p = p(t) \), where \( t \) denotes time. The external radius of the cylinder will be denoted by \( b \), the thickness of the elastic shell by \( h \), and the internal radius by \( a(t) \), assumed to be a function of time, but so that \( da/dt > 0 \). The cylinder is assumed to rotate with an angular velocity \( \omega \). In the more general case it can be assumed that the cylinder is subjected to the action of a nonhomogeneous non-stationary thermal field, and that the material properties of the cylinder and the elastic shell depend on the temperature. Let us assume that the cylinder is subjected to the conditions of a plane deformed state, i.e., \( \varepsilon_z = 0 \).

The given problem has already been examined \([1, 2]\) with this formulation, using the established theory of viscoelasticity as the basis. In this article, the solution of this problem is based on the nonlinear equation of creep \([3]\), generalized for the case of a complex stressed state, making it possible to describe the region of stable and unstable creep, up to the point of rupture.

Let us introduce the following nomenclature: \( \sigma_r \), \( \varepsilon_r \) and \( \sigma_\theta \), \( \varepsilon_\theta \), to denote, respectively, the radial and circumferential stresses and strains; \( E(T(t)) \), \( \nu_1 \) and \( \alpha_1 \) to indicate elastic constants and the thermal coefficient of expansion of the cylinder material; \( E_{sh}(T(t)) \), \( \nu_{sh} \) and \( \alpha_{sh} \) denoting the corresponding symbols for the thin elastic shell, assuming a uniform distribution of the temperature over the shell thickness.

Fig. 1. Variation of elastic modulus \( E_1 \), strength \( \sigma_B \), and stable-creep strength \( \sigma_s \) of the cylinder material, with respect to temperature.

Fig. 2. Variation of temperature, pressure, and internal radius, with respect to time.

For the general case we will assume that when the loading is applied, elastic and creep strains are induced in the material

\[ \varepsilon = \varepsilon_{el} + \varepsilon^c, \]

where \( \varepsilon^c \) is the creep strain. For the cylinder, the elastic strain is described by Hooke's law:

\[ \begin{align*}
\varepsilon_r &= \frac{1}{E_1} \sigma_r - \frac{v_1}{E_1} (\sigma_0 + \sigma_z) - \alpha_i T; \\
\varepsilon_\theta &= \frac{1}{E_1} \sigma_\theta - \frac{v_1}{E_1} (\sigma_0 + \sigma_r) - \alpha_i T; \\
\varepsilon_z &= \frac{1}{E_1} \sigma_z - \frac{v_1}{E_1} (\sigma_0 + \sigma_r) + \alpha_i T.
\end{align*} \]

The creep strain is described by the nonlinear equation

\[ \frac{d \varepsilon^c}{dt} = \frac{3}{2} \frac{\dot{\sigma}_1}{\sigma_1} \left[ C_0 e^{\alpha_0 + \beta T} + P_0 e^{-\lambda T} \varepsilon^c + q \varepsilon^c \right]. \]

where \( \varepsilon^c \) is the rate of creep strain; and \( \sigma_1 \) is the stress rate. The parameters \( C_0, \eta, k, P_0, \lambda, \alpha, \beta \) are determined on the basis of experimental investigations for creep, for a uniaxial stressed state. Based on Eqs. (1) and (2), we have the following expressions for the general strains of the cylinder

\[ \begin{align*}
\varepsilon_r &= \frac{1 - v_1^2}{E_1} \sigma_r - \frac{v_1 (1 + v_1)}{E_1} \sigma_0 + (1 + v_1) \alpha_i T + \varepsilon^c; \\
\varepsilon_\theta &= \frac{1 - v_1^2}{E_1} \sigma_\theta - \frac{v_1 (1 + v_1)}{E_1} \sigma_0 + (1 + v_1) \alpha_i T + \varepsilon^c; \\
\varepsilon_z &= \frac{1 - v_1^2}{E_1} \sigma_z - \frac{v_1 (1 + v_1)}{E_1} \sigma_0 + (1 + v_1) \alpha_i T + \varepsilon^c.
\end{align*} \]

For the case of a plane strained state, with \( \varepsilon_z = 0 \), we obtain from the relationships of Eq. (4)

\[ \begin{align*}
\varepsilon_r &= \frac{1 - v_1^2}{E_1} \sigma_r - \frac{v_1 (1 + v_1)}{E_1} \sigma_0 + (1 + v_1) \alpha_i T + \varepsilon^c; \\
\varepsilon_\theta &= \frac{1 - v_1^2}{E_1} \sigma_\theta - \frac{v_1 (1 + v_1)}{E_1} \sigma_0 + (1 + v_1) \alpha_i T + \varepsilon^c.
\end{align*} \]

Solving the relationships of Eq. (4), with respect to the stresses, we find

\[ \begin{align*}
\sigma_r &= \frac{F_1}{(1 - v_1) (1 - 2v_1)} [(1 - v_1) \varepsilon_r + v_1 \varepsilon_\theta - (1 + v_1) \alpha_i T - (1 - v_1) \varepsilon^c - v_1 (\varepsilon_0^c + \varepsilon^c)]; \\
\sigma_\theta &= \frac{F_1}{(1 - v_1) (1 - 2v_1)} [(1 - v_1) \varepsilon_\theta + v_1 \varepsilon_r - (1 + v_1) \alpha_i T - (1 - v_1) \varepsilon^c - v_1 (\varepsilon_0^c + \varepsilon^c)]; \\
\sigma_z &= \frac{F_1}{(1 + v_1) (1 - 2v_1)} [v_1 (\varepsilon_r + \varepsilon_\theta) - (1 + v_1) \alpha_i T - (1 - v_1) \varepsilon^c - v_1 (\varepsilon_0^c + \varepsilon^c)].
\end{align*} \]

Proceeding from Eqs. (5) and (6), the expressions for the rate of creep strain and of stresses will have the form

\[ \begin{align*}
\varepsilon^c_r &= \frac{2}{9} [(\varepsilon_0^c - \varepsilon^c)^2 + (\varepsilon^c - \varepsilon_0^c)^2 + (\varepsilon^c - \varepsilon^c)^2]; \\
\varepsilon^c_\theta &= \frac{1}{2} [(\sigma_r - \sigma_0)^2 + (\sigma_\theta - \sigma_0)^2 + (\sigma_z - \sigma_0)^2].
\end{align*} \]

Let us present the expressions for strain, in terms of the permutations

\[ \varepsilon_r = \frac{d u_r}{d t}; \quad \varepsilon_\theta = \frac{u_\theta}{r}. \]

The equation for the consistency of strains will be written as:

\[ \frac{d \varepsilon_{\theta}}{d r} + \frac{\varepsilon_\theta - \varepsilon_r}{r} = 0, \]

and the equilibrium equation, by