COMPUTER ANALYSIS OF PRIMARY CREEP CURVES ON THE BASIS OF THE McVETTY—GAROFALO—DAVIES EQUATION

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1. The practical importance of computer analysis of creep test results is obvious. Solution of this problem will increase the efficiency and quality of scientific research [1]. The problem is also of theoretical significance, since an objective analysis of the time dependence of creep strain makes it possible to obtain important information on the mechanism of thermally activated plastic deformation.

The primary results of creep tests are usually a discrete set of strain values \( \varepsilon_i \) (or elongation) for several moments in time \( t_i \) \((1 \leq i \leq N\), where \( N \) is the number of observations). In this case the equation of the creep curve takes the form

\[
\varepsilon = f(t, a_1, \ldots, a_n),
\]

where \( a_1, \ldots, a_n \) are several parameters depending on the stress \( \sigma \) and temperature \( T \). The primary results are always treated in two stages. In the first stage parameters \( a_1, \ldots, a_n \) are found for each of the creep curves, i.e., for each pair of values \( \sigma \) and \( T \) at which tests were made. These data are necessary for the second stage of the treatment — for example, to determine the creep strength and long-term strength or activation energy and the activation volume.

The second stage of the treatment will not be considered here. Let us note only that computer methods are rather widely used (see [2], for example). However, calculations are generally made by hand in the first stage. This requires large amounts of time, may involve subjective errors in the results obtained, and does not permit standardized determination of the creep parameters. Besides, only a small part of the useful information contained in the primary data is generally extracted from them, since otherwise the bulk of the calculations or graphic work becomes insurmountably large. These difficulties can be eliminated only by use of the computer.

A method of computer analysis of creep curves was described earlier [3]. However, the algorithm used, combining the method of least squares with the method of successive approximations, led occasionally (although rarely) to failures, which may be due to the divergence of the series of successive approximations. The problem of computer analysis was also raised in [1], but only with regard to a partial solution. In this work we attempt a more precise definition of the problem and propose a new algorithm to determine the parameters of Eq. (1).

![Fig. 1. Geometric meaning of the parameters of the McVetty—Garofalo—Davies equation and several other characteristics of creep.](image)


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Development of a method for computer analysis of the primary results of creep tests requires the solution of two interrelated problems: 1) the selection of a specific type of equation (1) to describe the creep curve; 2) the development of an algorithm and program to determine parameters $a_1, \ldots, a_n$ entering into this equation.

A valid selection of Eq. (1) is possible, strictly speaking, only on the basis of the creep theory. The most promising results are given by the formal activation theory of the time dependence of creep [4, 5]. It permits Eq. (1) to be written in the form

$$\varepsilon(t) = \varepsilon_0 + \sum_{i=1}^{n} \varepsilon_{im}^{(i)} \left[ 1 - \exp \left( -\frac{t}{\tau_{im}^{(i)}} \right) \right] + \varepsilon_{st} t + \sum_{k=1}^{n} \varepsilon_{sk} \exp \left( \frac{t - \theta}{\tau_{sk}} \right),$$

where $\varepsilon_0$ is the total elastic and plastic deformation occurring instantaneously under load. The terms in the right side describe transient, steady, and accelerating creep respectively. The first term on the right is based on data from [4], according to which transient creep can be represented in the form of the superposition of exponential inputs, "creep modes" (for our purposes it is expedient to consider a discrete spectrum); $\tau_{im}^{(i)}$ is the lag time of the $i$-th mode of transient creep; $\varepsilon_{im}^{(i)}$ is the maximum strain associated with it; $\varepsilon_{st}$ is the rate of steady creep.

In the framework of the formal activation theory the first two terms are also obtained in [5], but without taking the spectrum into account, i.e., only one mode of transient creep with maximum time $\tau_{1,\text{max}}^{(1)} = \tau_1$ is considered. Obviously, the last term in Eq. (2) can be described from the viewpoint of the formal activation theory, although it has not been done so far. These terms in Eq. (2) are written by analogy with the first term, and values $\tau_{j}^{(k)}$ and $\varepsilon_{im}^{(k)}$ have the same meaning as $\tau_{1}^{(1)}$ and $\varepsilon_{im}^{(1)} (\theta$ is the duration).

Equation (2) contains $2(1+n_1+n_2)$ parameters $\varepsilon_0$, $\tau_{1}^{(1)}$, $\varepsilon_{im}^{(1)}$, $\tau_{j}^{(k)}$ and $\varepsilon_{im}^{(k)}$ to be determined. If we limit ourselves to the simplest case, $n_1 = n_2 = 1$, retaining only terms with $\tau_{1,\text{max}}^{(1)} = \tau_1$ and $\tau_{j,\text{max}}^{(k)} = \tau_3$, then we obtain the McVetty — Garofalo — Davies equation [6-8]

$$\varepsilon(t) = \varepsilon_0 + \varepsilon_{im} \left[ 1 - \exp \left( -\frac{t}{\tau_1} \right) \right] + \varepsilon_{st} t + \varepsilon_{sm} \exp \left( \frac{t - \theta}{\tau_3} \right).$$

(The last term in the equation differs somewhat from that in [7, 8], which was explained in detail earlier [3].) Formula (3) contains six parameters. Let us note that $\varepsilon_0 > \varepsilon_0^*$, since $\varepsilon_0$ includes the strain of "exhausted" short-lived modes of transient creep. The meaning of the parameters considered above is illustrated in Fig. 1. A series of other values is also defined there, the meaning of which is accepted without explanation. Due to the inclusion of brief modes of transient creep in the initial strain curve described by Eq. (3), with $t \to 0$ the curve is extrapolated at point A, which is located above the initial point A' for the actual curve.

Tests have shown [4] that this deviation takes place in a section comprising approximately one-sixth the duration of the entire transient stage. A deviation from the real curve would be expected also with $t \to \theta$ at the "agonic" section of the creep curve ($\theta$ is time to failure). Limiting ourselves to the simplest form of