USE OF LINEAR MECHANICS OF DAMAGE THEORY IN THE DETERMINATION OF TURBINE ROTOR STRENGTH

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In the last two decades a great deal of work has been done on the brittle failure of constructions, including the rotors of turbogenerators. Using standard methods of studying material, employing impact specimens designed by Menagy and Charpy, showing the different tendency of the material for brittle destruction in certain conditions, it is unfortunately impossible to obtain quantitative data for predicting the limiting strength of the construction as a function of the properties of the metal, the geometry of the component, the presence of possible defects, and the existing stress field.

The present paper attempts to use the linear mechanics of damage to analyze the maximum strength of a welded rotor of the K-220-44 turbine made from steel grade 34KhMA.

Using the linear mechanics of damage it is possible to obtain a mathematical relationship between the geometry of the component, the dimensions of the crack or equivalent defect, to properties of the material, the stress field, and the so-called critical coefficient of stress intensity $K_{IC}$ (or the related magnitude of the destructive strength $G_{IC}$).

Knowing the critical coefficient of the stress intensity $K_{IC}$ of the material it is possible to find the stresses which lead to brittle failure of the rotor in the presence within it of a crack-like defect of a certain dimension, or to resolve the reverse problem: to determine the dimensions of the crack leading to brittle destruction of the rotor in certain conditions of loading (revolutions of the rotor).

If we know the magnitude $K_{IC}$ of the material it is possible to determine the critical dimensions of the cracks in the elements of the rotor. The critical dimension of the round plane of the crack in the center of the large solid disc we find according to the Sneddon formula [1] which is valid for bodies of infinite dimensions:

\[ C_{cr} = \frac{\pi K_{IC}^2}{4\sigma^2} \]  

where $C_{cr}$ is the radius of the round crack of critical dimension and $\sigma$ is the stress in the center of the disc.

In order to determine the height of the straight-through crack in the center of the solid disc we use the Irwin equation [1]

\[ C_{cr} = \frac{b}{\pi} \arctg \frac{K_{IC}^2}{\sigma b} \]  

where $b$ is the external diameter of the disc, and $C_{cr}$ is half of the height of the crack.

For cylindrical shells of the welded rotor located between the discs, the height of the axial semieliptical crack of the critical dimensions, protruding from the internal or external surface, can be calculated from the Irwin equation [1]

\[ C_{cr} = \frac{K_{IC}^2 Q}{1.21 \pi \sigma^2} \]  


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where $Q = \phi^2 - 0.212(\sigma/\sigma_0)^2$ is the parameter of the crack form. The latter is given in Fig. 1 as a function of the ratio $a/2c$ ($a$, $c$ are semilengths of the small and large diameters of the ellipse; $\sigma$ is the acting stress).

The critical depth of the annular crack located in the radial–peripheral direction and protruding from the internal or external surface of the cylindrical shell can be determined from the Buchner equation [2] which is deduced for notched bent specimens.

Let the cylindrical shell of the welded rotor consist of separate bent specimens joined together. In this case the height of the annular crack of the cylindrical shell, determined from the Buchner equation will be exaggerated, which contributes to enhanced reliability:

$$K_{1e} = \frac{6M}{(d-c)^{3/2}} \left[ f\left(\frac{c}{d}\right) \right]^{1/2},$$

where $c$ is the depth of the crack, $d$ is the thickness of the cylindrical shell, and $M$ is the bending moment.

The function $f(c/d)$ is given in Fig. 2. Let us convert the Buchner equation into the form

$$f\left(\frac{c}{d}\right) = \frac{K_{1e}d^3}{36M^2}.$$  

The first part of this expression is calculated for different values of $c/d$, and is compared with the curve $f(c/d)$ in Fig. 2. The intersection of the two curves gives the sought-after value $c/d$, and consequently $c_{cr}$.

One of the important questions determining the possibility of failure of the construction is the change in the dimension of the defects during operation. Ideas exist about the evaluation of the subgrowth of the crack to the critical dimensions during variable loading of the rotor in the turbine when stopping and starting.

An assessment of the number of starts during which the crack or crack-like defect would grow to a critical dimension can be made from the Wilson formula [3]

$$N = \frac{2}{(n-2)c_0M^n\Delta\sigma} \left[ \frac{1}{c_{cr}^2} - \frac{1}{c_{cr}^2} - \frac{1}{c_{cr}^2} \right],$$

for $n \neq 2$;

$$N = \frac{1}{c_0M\Delta\sigma} \ln \frac{c_{cr}}{c_1},$$

for $n = 2$,

where $N$ is the number of cycles leading to the subgrowth of the crack to the critical dimensions; $c_1$ and $c_{cr}$ are the initial and critical dimensions of the crack; $n$, $c_0$ are coefficients for the dependence of the rate of growth of the crack and the intensity of the stress $dc/dN = c_0\Delta\sigma$ (Fig. 3); $M$ is the geometrical parameter in the general formula $K = \sigma \sqrt{Mc}$.

The deduced equation is valid when the connection between the applied load, the dimension of the crack, and the coefficient of intensity of the stress is expressed by the equation $K = \sigma \sqrt{Mc}$. Furthermore,