A NONDESTRUCTIVE TESTING METHOD FOR DETERMINING THE STRESS-STRAIN RELATIONSHIP IN MATERIALS

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Engineering applications using components manufactured out of powdered metals are finding increasingly widespread use. This method allows one to meet the required strength and ductility of manufactured products provided the correct treatments and initial materials are selected. However, during production, scatter in the mechanical properties may arise which is hard to evaluate using selective mechanical testing. Therefore, the problem amounts to determining the individual properties of each component for complete inspection of the manufactured products. This problem can be solved using nondestructive testing techniques.

The simplest of these is hardness inspection of components which is widely used in machine engineering. However, during traditional hardness testing the characteristic determined is the resistance of the material to elastic-plastic indentation of the material which is not directly related to the elastic modulus, yield point, or other mechanical properties. For certain materials a correlation has been established relating the hardness and several mechanical properties, but they have limited applications [1].

Currently, widespread use is made of the technique of constructing hardness diagrams from elastic and plastic impression tests using a spherical indentor. The hardness diagrams, which have been constructed using a number of specific coordinates, have a qualitative association with tensile test diagram. It would be very useful to further study the hardness diagrams with the aim of reconstructing them in terms of the more generalized stress-strain curve.

The manner in which material deformation curves can be obtained from the results of indentor impression tests, as presented earlier [3, 4], is as follows. From the experimentally determined penetration depth \( \delta \) of the indentor as a function of the applied load \( P \) a solution to the elastic-plastic contact problem arises which can be used to establish a concrete association between the stress intensity \( \sigma_i \) and the strain \( \varepsilon_i \) in such a manner that the solution satisfies the experimentally determined function \( P = P(\delta) \).

This method of determining the stress-strain relationship of materials allows one to identify the mechanical properties both in the elastic and plastic regimes based on the
actual strength of the material without having to resort to correlated relationships. The
required initial information comes from the $P = P(\delta)$ diagram which can be obtained experi-
mentally.

Assuming an absence of mass forces and that the amount of deformation is infinitely
small, we can obtain the following system of equations for determining the stress-strain
relationship from the results of indenter tests:

\begin{align*}
equilibrium equations & \quad \sigma_{ij,j} = 0; \\
Cauchy's formula & \quad \varepsilon_{ij} = 1/2(u_{i,j} + u_{j,i}); \\
formuli obtained from the theory of small elastic-plastic strains & \quad \varepsilon_{ij} - \delta_{ij} \varepsilon_0 = 3/2 \frac{s_i}{\alpha_i} [\sigma_{ij} - \delta_{ij} \sigma_0]; \\
& \quad \sigma_0 = 3K \varepsilon_0; \\
& \quad \sigma_i = f(\varepsilon_i); \\
boundary conditions outside the contact zone & \quad \tau = 0, \sigma_n = 0, u_i = 0; \\
and the boundary conditions inside the contact zone & \quad \sigma_i = -p(r) \leq 0; \\
& \quad \int p(r) d\Omega = P; \\
& \quad \delta = \delta_0.
\end{align*}

where $\sigma_{ij}$ and $\varepsilon_{ij}$ are the stress and strain tensors, respectively; $u_i$ is the displacement
vector; $\sigma_n$ is the normal stress; $\tau$ is the shear stress; $p(r)$ is the contact pressure; $\Omega$ is
the area of contact; $P$ is the load applied to the indenter; $\varphi(r, z)$ is the indenter surface
equation; $\delta$ is the calculated indenter displacement; and $\delta_0$ is the experimentally determined
displacement of the indenter at a load of $P$. The $z$ axis is the axis of symmetry and the
direction of penetration.

In contrast to the formulation of direct problems, Eq. (5) is unknown in this case and
must be found by solving the boundary condition (8).

At this time an analytical method for solving a symmetrical elastic-plastic contact
problem with a variable contact boundary without a knowledge of the strength relationship
(5) has not been worked out. Therefore, in what follows we will solve it using the method
of finite elements (MFE) [5]. In order to solve for the plastic regime a method of variable
elastic parameters is used and Eqs. (3)-(5) are put in the form of a generalized Hooke's law
where the parameters $E^*$ and $v^*$ are functions of the stress state at a point and therefore vary
from point to point in the body [6]:

\begin{equation}
E^* = E^*(\sigma_i/e_i); \quad v^* = v^*(\sigma_i/e_i).
\end{equation}

In turn, the ratio $\sigma_i/e_i$, which is determined from the strain curve (5), appears as a
function of the experimentally determined indenter displacement $\delta$. Consequently, the stress-
strain relationship in the material of interest may be represented as:

\begin{equation}
\{\sigma\} = [D]^p\{\varepsilon\},
\end{equation}

where the matrix

\begin{equation}
[D]^p = [D (\sigma_i/e_i (\delta))]
\end{equation}
in the case of axial symmetry has the form