

In determining the temperature fields and thermal stresses in plates and shells, it is usually assumed that the temperature $T$ is a linear function of the coordinate $x^3$. If the heat-exchange conditions are such that the temperature variation rate is very high, the above assumption is inadmissible. In this case, a sharp change in temperature during the thermal shock occurs only in a very thin boundary layer next to the surface, and the temperature vs $x^3$ curve is essentially nonlinear.

In rapid thermal expansion, the dynamic effects caused by the movement of the medium in the boundary layer preclude the applicability of the classical shell theory. Such problems can be solved conveniently by using the more accurate equations of shell theory [1, 2]. They are obtained from the elastic-theory relationships by applying to the latter the projection method for reducing their dimension. In our case, the dynamic thermoelasticity equations describing the behavior of the shell in time are

$$
\nabla_n \sigma^{i1} - (2n + 1) \sum_{k=0}^n h_{2k+2}^{n-2k-2} \sigma^{i3} \frac{1}{R} \frac{\partial}{\partial x^3} \sum_{i=0}^n h_{2k}^{n-2k-1} \sigma^{i3} - \frac{1}{R} \frac{\partial}{\partial x^3} \sum_{i=0}^n h_{2k+2}^{n-2k-2} \sigma^{i3} = \rho \frac{\partial^2 u^i}{\partial t^2};
$$

$$
\nabla_n \sigma^{i2} - \frac{1}{R} \sigma^{i3} - (2n + 1) \sum_{k=0}^n h_{2k}^{n-2k-1} \sigma^{i3} \frac{1}{R} \frac{\partial}{\partial x^3} \sum_{i=0}^n h_{2k+2}^{n-2k-2} \sigma^{i3} = \rho \frac{\partial^2 u^i}{\partial t^2};
$$

$$
\nabla_n \sigma^{i3} - R \sigma^{i3} - (2n + 1) \sum_{k=0}^n h_{2k}^{n-2k-1} \sigma^{i3} \frac{1}{R} \frac{\partial}{\partial x^3} \sum_{i=0}^n h_{2k+2}^{n-2k-2} \sigma^{i3} = \rho \frac{\partial^2 u^i}{\partial t^2};
$$

$$
\frac{\partial q^i}{\partial x^3} - (2n + 1) \sum_{k=0}^n h_{2k}^{n-2k-1} q \frac{1}{R} \frac{\partial}{\partial x^3} \sum_{i=0}^n h_{2k+2}^{n-2k-2} q = \rho \frac{\partial^2 u^i}{\partial t^2} + (n + \frac{1}{2}) h_{n+1}^{n+1} \left( \left( \frac{1}{R} \frac{\partial}{\partial x^3} \right) \left( - \alpha_+ \phi_+ \right) - \left( - 1 \right)^n \left( \frac{1}{R} \frac{\partial}{\partial x^3} \right) \left( - \alpha_- \phi_- \right) \right) = - c \frac{\partial T}{\partial t}, \quad (i = 1, 2; \; n = 0, 1, 2, \ldots, N; \; \sigma^{i1} = 0 \quad \text{for} \quad r < 0)
$$

where $R$ is the shell radius; $2h$ is the thickness; $c$ is the volume specific heat of the shell material; $\rho$ is the density; $u^i$, $\sigma^{ij}$, and $q^i$ are the moments of the contravariant components of the elastic displacement vector, the stress tensor, and the heat flux vector, respectively; and $T$ are the moments of the temperature function. The symbols $+$ and $-$ denote the heat-transfer coefficients and temperatures of the medium at the outer and inner surfaces, respectively. A right-hand coordinate system is used: $x^1$ is oriented along the generatrix; $x^2$ along a circular arc; and $x^3$ is normal to the surface. The system of Legendre polynomials $P_n(x^3/h)$ is used as the basis. Using the laws of Hooke and Fourier, we represent the values $\sigma^{ij}$ and $q^i$ in terms of moments of the components of the strain tensor $e^{ij}$ and the temperature moments $T$:

$$
\sigma^{i1} = \lambda \phi^{i1} + 2\mu e^{i1} - 3\alpha_1 \left( \lambda + \frac{2\mu}{3} \right) T e^{i1};
$$

$$
\sigma^{i2} = \lambda \phi^{i2} + 2\mu e^{i2} - 3\alpha_2 \left( \lambda + \frac{2\mu}{3} \right) T e^{i2};
$$

$$
\sigma^{i3} = \lambda \phi^{i3} + 2\mu e^{i3} - 3\alpha_3 \left( \lambda + \frac{2\mu}{3} \right) T e^{i3};
$$

$$
q^i = \rho_0 \phi^i + \rho \frac{\partial T}{\partial x^3} \left( \left( \frac{1}{R} \frac{\partial}{\partial x^3} \right) \left( - \alpha_+ \phi_+ \right) - \left( - 1 \right)^n \left( \frac{1}{R} \frac{\partial}{\partial x^3} \right) \left( - \alpha_- \phi_- \right) \right).
$$


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Fig. 1. Time variation of the temperature along the shell thickness.

Fig. 2. Temperature variation at the heating surface.

\[ \sigma_{33} = \frac{\lambda}{r_0^2} \dot{\theta} + 2\mu \epsilon_{33}^2 - 3\alpha_0 \left( \frac{\lambda + 2\mu}{3} \right) \frac{n}{r_0}; \]

\[ \sigma_{22} = \frac{\lambda}{r_0^2} \dot{\theta} + 2\mu \epsilon_{22}^2 - 3\alpha_0 \left( \frac{\lambda + 2\mu}{3} \right) \frac{n}{r_0}; \]

\[ \sigma_{33} = \lambda \dot{\theta} + 2\mu \epsilon_{33}^2 - 3\alpha_0 \left( \frac{\lambda + 2\mu}{3} \right) \frac{n}{r_0}; \]

\[ q^i = -\lambda_0 \frac{\partial T}{\partial x_i}; \]

\[ q^3 = -\lambda_t (2n + 1) \sum_{h=0}^{\infty} h^{-2h+2n+1} \frac{n}{T}; \]

\[ T^+ = \sum_{h=1}^{\infty} h^{-h-1} \frac{n}{T} \]

\[ T^- = \sum_{h=1}^{\infty} h^{-h-1} \frac{n}{T} \]

where \( \theta \) is the moment of the first invariant of the strain tensor; \( g^{ij} \) are the contravariant components of the metric tensor; \( \lambda \) and \( \mu \) are the Lamé coefficients; \( \lambda_t \) is the coefficient of thermal conductivity; and \( \alpha_0 \) is the coefficient of linear thermal expansion.

We express the values of \( e^{ij} \) and \( \theta \) in terms of the moments \( u^1 \) and \( u^2 \) of the displacement vector:

\[ e_{11} = \nabla u^1; \quad e_{12} = \frac{1}{2} \left( g^{ij} \nabla u^2 + \frac{1}{R_0} \nabla u^1 \right); \]

\[ e_{13} = \frac{1}{2} \nabla u^1 + \left( n + \frac{1}{2} \sum_{h=0}^{\infty} h^{-2h-2} \frac{n^{h+2n+1}}{u^1} \right); \]

\[ e_{22} = \nabla u^2; \quad e_{23} = \frac{1}{2} \left( \frac{1}{R_0} \nabla u^3 + \nabla u^2 \right); \]

\[ e_{33} = (2n + 1) \sum_{h=0}^{\infty} h^{-2h-2} \frac{n^{h+2n}}{u^3}; \]

\[ \theta = g_{11} e_{11} + R e_{22} + e_{33}. \]

By substituting (2) and (3) in relationship (1), we obtain the resolvents with respect to \( u^1 \) and \( T \).

By integrating this system of equations, we find the displacement and temperature moments. The temperature and the components of the elastic displacement vector are calculated by means of the expression

\[ u^i = \sum_{n=0}^{\infty} h^{-n-1} u^n \frac{P_n(1/2)}{r_0}; \quad T = \sum_{n=0}^{\infty} h^{-n-1} T P_n \left( \frac{r^2}{h} \right). \]

In practical calculations, the dimension \( N \) of the basis \( P_n(x^2/h) \) is assumed to be limited.

Since the projection operation does not alter the characteristics of the initial system of equations, relationships (1), which describe the motion of the shell and the evolution of the thermal field, have preserved their hyperbolic and parabolic characteristics. This allows us to investigate on the basis of